On efficiency of multi-stage channel with bargaining over wholesale prices

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Abstract

This paper considers a multi-stage channel with deterministic price-sensitive
demand. Two systems for pricing decisions, i.e., the bargaining system and
the leader-follower system, are compared. We characterize the necessary and
sufficient conditions on the power structure, under which the solution of the
bargaining system Pareto dominates that of the leader-follower system. Also,
under such conditions, we give a tight upper bound of channel efficiency of
the bargaining system, which converges to 100% channel efficiency as the
number of stages increases to infinity.

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1. Introduction

The interaction among channel members is one of the major factors that affects channel performance. So far, various contracts are commonly used between channel members, such as wholesale price contracts, two-part tariff contracts, etc. Most channel models on contracts focus on the leader-follower system, in which the parameters of a contract are unilaterally determined by the contract leader. However, other than the unilateral scenario, some empirical evidences suggest that bilateral bargaining exists between upstream and downstream members (see, e.g., [6], [7] and [11]). Here, we quote Ertel’s statement in [7] to emphasize this point:

“Every company today exists in a complex web of relationships, and the shape of that web is formed, one thread at a time, through negotiations. Purchasing and outsourcing contracts are negotiated with suppliers. Marketing arrangements are negotiated with domestic and foreign distributors. The contents of product and service bundles are negotiated with customers. Product development pacts are negotiated with joint-venture partners.”

Hence, a member in a channel may face various negotiations, and may play different roles in different negotiations. However, the existing literature that considers channel negotiations mostly focuses on two-stage channels where a member in the channel only plays the seller role or the buyer role in the negotiations. Therefore, how a member’s decision is affected by playing different roles in different negotiations and how the whole channel perfor-
mance is affected by these negotiations are not discussed in previous studies. In this paper, we try to fill the gap. To do so, we analyze the bargaining system for a multi-stage serial channel model, in which each pair of upstream and downstream members bargains over the unit wholesale price. More specifically, the Nash solution is considered as the result of the negotiations. The negotiations take place sequentially from upstream to downstream, which coincides with many existing multi-stage channel scenarios (see, e.g., [4], [5] and [13]) that non-negotiation decisions (but leader-follower decisions) are made sequentially from upstream to downstream. Under this setting, a member in the channel plays the buyer role when he bargains with his upstream member, and plays the seller role when he bargains with his downstream member. From our study, we can see that multi-stage channels may behave very differently from two-stage channels. For example, we show that in a two-stage channel, when the leader-follower system changes to a bargaining system, the supplier loses and the retailer gains. However, in a multi-stage channel (with at least three stages), we show that it is possible that all members gain when the leader-follower system changes to a bargaining system.

The literature on bargaining in two-stage channels can be generally divided into two streams. One stream deals with the problem of allocating the additional profits received through channel coordination (see, e.g., [10] and [12]). The other stream discusses channel decision making through a bargaining process (see [1] and [16] for surveys), while the whole channel may not reach coordination. We are interested in the second stream, and to the best of our knowledge, [11] is the only reference closely related to our work in this stream. In [11], Iyer and Villas-Boas assume demand is stochastic
and price sensitive. They show that a more powerful retailer can coordinate the channel while a more powerful supplier can increase the effect of double marginalization; in addition to this, a powerful retailer does not necessarily harm the manufacturer’s profit. In our work, we confirm that a more powerful downstream member can coordinate the channel while a more powerful upstream member can increase the effect of double marginalization [11]; in addition to this, in the two-stage channel, a powerful retailer does harm the manufacturer’s profit, which differs from the result of [11]. The reason of such difference is as follows: Iyer and Villas-Boas build a simultaneous model, i.e., the negotiated wholesale price and the retail price set by the retailer form a simultaneous Nash equilibrium; while in our work, we assume a sequential process, i.e., the choice of the retail price is based on the negotiated wholesale price. In Iyer and Villas-Boas’ simultaneous model, the negotiated wholesale price and the retail price are determined at the same time. If the upstream member have a strong power in the negotiation, then two channel members may face intense competition in the simultaneous Nash game, hence both members will gain very little. In this case (i.e., upstream member have a strong power in the bargaining), increasing in the bargaining power of the downstream member can mitigate the competition in the simultaneous Nash game (since the downstream member have more impact in the negotiation), hence can benefit both channel members. However, in our sequential model, stackelberg leader always have the first-move advantage, and the downstream member can hardly compete with the leader as a follower. Hence, increasing in the bargaining power of the downstream member always harms the upstream member.
In [14], Lovejoy considers a bargaining process in a multi-stage channel with multiple competing members at each stage, in which demand is fixed and a static bargaining solution is discussed. In contrast, we consider demand to be price-sensitive and study a dynamic bargaining solution (i.e., our paper and Lovejoy’s paper have different assumptions on the order of the negotiations). Hence, efficiency is not a consideration in Lovejoy’s model (his channel is always coordinated with 100% efficiency), whereas in our model channel efficiency is a major issue because the power structure has impact on it. Nguyen also discusses a bargaining process in a multi-stage channel with multiple competing members at each stage in [19]. He assumes there is one type of indivisible good in the channel, and a dynamic repeated bargaining process is studied. Nguyen mainly studies the stationary equilibria and the limit behavior of these equilibria as the population of agents at each stage goes to infinity. Different from this work, we do not consider a dynamic repeated bargaining process, instead, we adopt Nash solution to statically characterize the bargaining outcome, and we mainly discuss the channel performance when the number of stages goes to infinity. There are also many other papers that study multi-stage channels, but all of them (see, e.g., [4], [5], [13] and [15]) do not take the bargaining process into account.

Comparing with existing results on multi-stage channels with or without bargaining processes, our study provides some new insights. For example, in our model, channel efficiency under the bargaining system always outperforms that under the leader-follower system. Furthermore, we obtain sufficient conditions under which the solution of the bargaining system Pareto dominates that of the leader-follower system. Also, over all power structures
under which the bargaining system Pareto dominates the leader-follower system, we characterize the power structure which optimizes channel efficiency. In addition, we show that channel efficiency under this best power structure converges to 100% as the number of stages increases to infinity. Based on these results, some valuable insights in member behavior are revealed. For instance, because the power structure affects channel performance and each member’s profit significantly, when a member needs to choose a channel cooperator, he should not only consider the cooperator’s power relative to himself but also should take the cooperator’s relationship with other members into account. In addition, provided a channel member tries to bargain with his immediate upstream member (if he has one) for a lower wholesale price, not only he, himself, but also the whole channel may benefit from his bargaining behavior.

The remainder of our paper is organized as follows. Section 2 gives detailed descriptions of the multi-stage channel, the channel efficiency and the bargaining process. In Section 3, a closed-form expression for the pricing decision at each stage is developed, together with the necessary and sufficient conditions under which the bargaining system Pareto dominates the leader-follower system. Section 4 extends the model and all results to a different demand model. Brief discussions and directions for future research are presented in Section 5.
2. Basic model and assumptions

In this section, the bargaining problem and the basic model of the multi-stage channel are formulated. Throughout the paper, the multi-stage channel has a single firm at each stage. Three types of channel systems (the coordinated system, the leader-follower system and the bargaining system) and associated channel efficiencies are defined and will be analyzed in Section 3.

2.1. Bargaining problem and Nash solution

The Nash solution of the bargaining problem was first proposed by Nash in [18], in which the two parties have the same bargaining powers. In the following, we introduce a generalized Nash solution, in which the two parties may have different bargaining powers.

For a bargaining problem with two parties, let $\theta_1 (\geq 0)$ and $\theta_2 (\geq 0)$ be the bargaining powers of the two parties, respectively. Denote by $(c_1, c_2)$ the disagreement point, i.e., $c_1$ and $c_2$ are the payoffs of both parties when they do not collaborate. Let a convex compact set $U \subset \mathbb{R}^2$ represent the set of all feasible payoffs of both parties when they collaborate. If there exists a point $(u_1, u_2) \in U$ such that $u_1 \geq c_1$ and $u_2 \geq c_2$, then the Nash solution is defined to be the following point:

$$(\pi_1, \pi_2) = \arg \max \left\{ (u_1 - c_1)^{\theta_1} \cdot (u_2 - c_2)^{\theta_2} \mid (u_1, u_2) \in U \right\}.$$ 

Formally, the payoffs of the two parties are $\pi_1$ and $\pi_2$ that maximize $(\pi_1 - c_1)^{\theta_1} \cdot (\pi_2 - c_2)^{\theta_2}$. More interpretations of the bargaining powers in the generalized Nash solution are discussed in [2].

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2.2. Leader-follower system

Our multi-stage channel configuration is similar to that in [13]. There are \( n \) members in the serial channel, denoted by \( P_1, \ldots, P_n \), with \( P_1 \) the most downstream firm. The transfer payment from a downstream member to his immediate upstream member is specified to a wholesale price only contract, i.e., the upstream member merely charges his immediate downstream member a fixed wholesale price per unit ordered. We focus on this simplest contract because it is commonly observed in practice with satisfactory qualities [3]. We assume that \( P_1 \) faces a deterministic demand, \( D(p_1) = (a + bp_1)^d \) (either \( d < -1, b > 0 \) and \( a \geq 0 \); or \( d > 0, b < 0 \) and \( a > -bc \)), which is known to all the channel members. We choose the demand function because commonly used two types of demand functions in the literature (linear, \( D(p_1) = a - bp_1, a > c \) and \( b > 0 \), see, e.g., [17], and iso-elastic, \( D(p_1) = bp_1^d, b > 0 \) and \( d < -1 \), see, e.g., [20]) are special cases of our demand.

Under the leader-follower system, the wholesale price \( p_i \) is unilaterally determined by \( P_i \) \((i = 1, \ldots, n)\). More specifically, member \( P_n \) produces a single product with unit cost of \( p_{n+1} = c \); we assume \( p_{n+1} = c \) to be common knowledge. This is reasonable since \( c \) is mostly related to the raw material, which is not difficult to know. \( P_n \) determines the wholesale price \( p_n \) first. For member \( P_i \), \((i \in \{n - 1, n - 2, \ldots, 2\})\), once the wholesale price \( p_{i+1} \) is given, he determines the order quantity \( q_i \) and the wholesale price \( p_i \). Finally, retailer \( P_1 \) faces a unit wholesale price \( p_2 \), and determines the order quantity \( q_1 \) and the retail price \( p_1 \). Therefore, each channel member can predict correctly how many products his immediate downstream member would order, i.e., \( q_{n-1} = \ldots = q_1 = D(p_1) \).
2.3. Bargaining system

Under the bargaining system, the wholesale price $p_i$ ($i = 2, \ldots, n$) is determined by the bargaining process between $P_i$ and $P_{i-1}$. We assume the relative bargaining powers of $P_i$ and $P_{i-1}$ are $\lambda_i$ and $1 - \lambda_i$ respectively, and the disagreement point is $(0, 0)$. Additionally, let $\lambda_1 = 1$ to simplify the notation. Denoted by $(\lambda_2, \lambda_3, \ldots, \lambda_n)$ the power structure of the channel. We assume that the power structure of the channel is common knowledge to all channel members. This assumption is reasonable since sometimes it is not difficult to know who has stronger power in the negotiation. For example, when a manufacturer bargains with Walmart, Walmart will have a very strong bargaining power in most cases. Since the power structure of the channel is common knowledge, channel members can predict correctly how much the final demand would be (i.e., $q_{n-1} = \ldots = q_1 = D(p_1)$), hence the order quantity is automatically determined after the wholesale price is negotiated.

Remark: In our model, we have made a strong assumption that the power structure is common knowledge. However, if a channel member knows very little of all his downstream members (hence, he does not know the power structure), we can make an alternative assumption such that our model analyses are still valid. That is, we assume the channel member is familiar with the demand he faces (i.e., the order quantities from his downstream member). He knows how the demand is affected by the wholesale price through maybe historical data. Hence, our model is suitable for channels that have run for a long time and may fail to characterize newly built channels.

Similar to the leader-follower system, the negotiations take place sequen-
tially from the upstream to the downstream, and demand is determined after all the pricing decisions are made. In this paper, the reason we do not consider other orders of negotiations is that, in the leader-follower system with a wholesale price contract, it is not suitable to consider other orders of decisions. Our main results are based on the comparisons between the bargaining system and the leader-follower system, hence it is better to keep the same order of decisions in two systems.

In our model, we assume that, when $P_i$ and $P_{i-1}$ negotiate over the wholesale price $p_i$, they both know the negotiated wholesale price $p_{i+1}$. Such an assumption is made for the following two reasons: the cost $c$ and the relative bargaining powers between each upstream and downstream member pair are assumed to be common knowledge, therefore $p_{i+1}$ can be correctly predicted; it is also possible that the wholesale prices are determined by an auction, and the results of the auction (including the wholesale price) are shown to the public in many auction mechanisms.

2.4. Coordinated system and channel efficiency

When there is a “centralized decision maker” who makes all the decisions to achieve the maximum profit for the whole channel, we call it a coordinated system. Channel efficiency of a system is usually defined as the ratio of the channel profit under the system (denoted by $\Pi$) to that under the coordinated system (denoted by $\Pi^C$), i.e.,

$$\eta = \frac{\Pi}{\Pi^C}.$$ 

So far, we have introduced the basic settings. Now, we list other notation that is used in the subsequent sections:
$p_k^B$: wholesale price offered by $P_k$ in the bargaining system.

($p_{n+1}^B = c$ is defined to simplify the notation.)

$\Pi_k^B$: profit gained by $P_k$ in the bargaining system.

$\Pi_k^L$: profit gained by $P_k$ in the leader-follower system.

$\Pi^B$: channel profit of the bargaining system.

$\Pi^L$: channel profit of the leader-follower system.

$\Pi^C$: channel profit of the coordinated system.

3. Comparing the bargaining and leader-follower systems

In this section, we first characterize the pricing decisions under the bargaining system. Then, we make a comparison between the bargaining and leader-follower systems. Finally, we analyze the channel efficiency.

3.1. Pricing decisions under the bargaining system

The following theorem characterizes all the pricing decisions under the bargaining system. (The proof of the theorem, and those of other theorems hereafter, are all presented in Appendices.)

**Theorem 1.** For $i = 1, 2, \ldots, n$, given $p_{i+1}$ such that $a + bp_{i+1} \geq 0$, the bargaining over $p_i$ results in $p_i^*\left(p_{i+1}\right)$, satisfying

$$(a + bp_i^*(p_{i+1})) = \left(1 - \frac{\lambda_i}{d + 1}\right)(a + bp_{i+1}) \geq 0.$$ 

**Remark:** If $p_{i+1}$ is chosen such that $a + bp_{i+1} < 0$ (which can only happen when $b < 0$), then demand is 0 and all members in the channel earn nothing. Since we are assuming $a + bp_{n+1} = a + bc > 0$ (when $b < 0$),
which means it is possible for all members in the channel to earn strictly positive profit, hence the pricing decisions will satisfy $a + bp_{i+1} \geq 0$.

In Theorem 1, we can see that the bargaining result of $p_i$ depends linearly on the relative bargaining power of $P_i$ to $P_{i-1}$. Furthermore, the stronger this relative power $\lambda_i$, the higher the wholesale price $p_i$.

Since $p_{n+1} = c$ is given to $P_n$, we have (for $k = 1, 2, \ldots, n$)

$$p_k^B = \left( \frac{a}{b} + c \right) \prod_{i=k}^{n} \left( 1 - \frac{\lambda_i}{d+1} \right) - \frac{a}{b}. \quad (1)$$

Then, the profit of $P_k$ is given by (for $k = 1, 2, \ldots, n$)

$$\Pi_k^B = \left( \frac{a + bc}{b} \right)^{d+1} \frac{-\lambda_k}{d+1} \prod_{i=1}^{n} \left( 1 - \frac{\lambda_i}{d+1} \right)^d \prod_{j=k+1}^{n} \left( 1 - \frac{\lambda_i}{d+1} \right) \prod_{j=1}^{n} \left( 1 - \frac{\lambda_i}{d+1} \right)^d. \quad (2)$$

Summing up over all $P_k$’s profits, the channel profit is

$$\Pi^B = \left( \frac{a + bc}{b} \right)^{d+1} \left( \prod_{i=1}^{n} \left( 1 - \frac{\lambda_i}{d+1} \right) - 1 \right) \prod_{i=1}^{n} \left( 1 - \frac{\lambda_i}{d+1} \right)^d. \quad (2)$$

Taking the derivative with respect to $\lambda_i$ ($i = 2, \ldots, n$) in (2), we find $\frac{\partial \Pi^B}{\partial \lambda_i} \leq 0$. This means, a more powerful downstream member in the pair of two neighboring members always promotes channel coordination. The reason is that the channel profit is only determined by the retail price $p_1$. Due to double marginalization, $p_1$ is larger than its optimal solution in the coordinated system. However, if the downstream member in the pair of two neighboring members has stronger relative bargaining power, he would get a better wholesale price, which eventually decreases the retail price $p_1$. This result coincides with that of the existing studies (see, e.g., [11]) for two stage channel, but our result is a little more general, saying that the above is true for channel with any number of stages.
Similarly, taking the derivative with respect to $\lambda_i$ ($i = 2, \ldots, n$) in (1), we find $\partial \Pi_k^B / \partial \lambda_k \geq 0$ and $\partial \Pi_k^B / \partial \lambda_i \leq 0$ ($k = 2, \ldots, n, k \neq i$). This means that, for a given $i$, $P_i$ prefers to have stronger relative bargaining power when bargaining with $P_{i+1}$ and $P_{i-1}$, and also prefers the downstream members having stronger relative bargaining power in all the other pairs of two neighboring members. The reason of the former finding is simply because stronger relative bargaining power usually implies larger share of the pie. The reason of the latter finding is because a more powerful downstream member eventually decreases the retail price $p_1$, and hence raises the demand.

The above results suggest that a downstream member should bargain over the wholesale price with his immediate upstream member whenever possible. The effort made by the downstream member in the bargaining not only increases his own profit but also promotes channel coordination. In addition to this, when a channel member needs to choose a cooperator, he should not only consider the cooperator’s power relative to himself but also should take the cooperator’s relationship with other channel members into account.

**Remark**: When $\lambda_k = 0$ for some $k \in \{2, \ldots, n\}$, we know from (1) that only $P_k$ obtains zero profit, which differs from [14] (see Proposition 7) that all upstream members ($P_n, \ldots, P_k$) obtain zero profit.

When $\lambda_i = 0$ for all $i = 2, \ldots, n$, the channel efficiency reaches 100%. But such coordination is not implementable since it is achieved at the expense of all the channel members except $P_1$ (i.e., $\Pi_k^B = 0$ for $k = 2, \ldots, n$). Consequently, a bargaining system which is implementable should satisfy the following condition: the bargaining system Pareto dominates the leader-follower system.
3.2. Profits comparison

If all bargaining power is given to the upstream member at each bargaining process, the bargaining system will result in the outcome of the leader-follower system. Therefore, by setting $\lambda_i = 1$ for $i = 2, \ldots, n$, we have from (1) and (2) that

$$\Pi_k^L = \frac{(a + bc)^{d+1}}{b} \frac{-1}{d+1} \left( \frac{d}{d+1} \right)^{nd+n-k},$$

$$\Pi^L = \frac{(a + bc)^{d+1}}{b} \left( \left( \frac{d}{d+1} \right)^n - 1 \right) \left( \frac{d}{d+1} \right)^{nd}.$$  

Since $\partial \Pi^B / \partial \lambda_i \leq 0$, we have $\Pi^B \geq \Pi^B |_{\lambda_i=1,i=2,\ldots,n} = \Pi^L$. That is, the channel profit is always higher in the bargaining system than that in the leader-follower system.

Combining (1) and (3), we get the necessary and sufficient condition, under which the bargaining system Pareto dominates the leader-follower system, as follows

$$1 \leq \lambda_k \prod_{i=1}^{n} \left( 1 + \frac{1 - \lambda_i}{d} \right)^d \prod_{j=k+1}^{n} \left( 1 + \frac{1 - \lambda_j}{d} \right), \quad k = 1, 2, \ldots, n. \tag{5}$$

Remark: Parameters $a$ and $b$ do not play a role in the above equations. The reason is that, the profit of the channel member $P_k$ is

$$\Pi_k^B = (p_k - p_{k+1})(a + bp_k)^d = \frac{d^{d+1}}{b} \left( \frac{b}{a} p_k - \frac{b}{a} p_{k+1} \right)(1 + \frac{b}{a} p_1)^d.$$

If $bp_k/a$ is regarded as the decision variable, then $a$ and $b$ are only constant multipliers. Hence $a$ and $b$ do not appear in (5).

For example, when $a > -bc$, $b < 0$, $d = 1$ and $n = 3$, the Pareto range of (5) is surrounded by two curves ($1 = \lambda_2(2 - \lambda_2)(2 - \lambda_3)^2$ and $1 = \lambda_3(2 - \lambda_2)(2 - \lambda_3)$), as shown in Figure 1(a), which is emphasized in gray.
For $a > -bc$, $b < 0$, $d = 1$ and $n = 4$, the Pareto range of (5) is surrounded by three surfaces $(1 = \lambda_2(2 - \lambda_2)(2 - \lambda_3)^2(2 - \lambda_4)^2$, $1 = \lambda_3(2 - \lambda_2)(2 - \lambda_3)(2 - \lambda_4)^2$ and $1 = \lambda_4(2 - \lambda_2)(2 - \lambda_3)(2 - \lambda_4))$, which is displayed in Figure 1(b).

The following theorem characterizes some properties of the inequalities in (5).

**Theorem 2.** For $n = 2$, we have $\Pi_L^2 \geq \Pi_B^2$ and $\Pi_L^1 \leq \Pi_B^1$.

For $n \geq 3$, there exist $0 \leq \alpha < \beta \leq 1$(that are independent of $n$), such that if $\lambda_i \in [\alpha, \beta]$ for all $i = 2, \ldots, n$, then (5) holds.

The first part of the above theorem indicates that when there are only two stages in the channel, the retailer will always benefit from bargaining while the manufacturer will always suffer from bargaining (which means that the bargaining system cannot strictly Pareto dominates the leader-follower system). However, when the channel has more than two stages, the bargaining system can dominate the leader-follower system for some cases. The reason is that, on one hand, more stages in the bargaining system make the profit allocation for all members more flexible (the power structure of a channel...
with $n$ stages is an $n - 1$ dimension variable), therefore give more chance for the bargaining system to Pareto dominate the leader-follower system. On the other hand, the channel efficiency of the leader-follower system is decreasing in the number of stages (which is shown in the next subsection). Therefore, as the number of stages increases, the profitability of the whole channel and all channel members decrease, which further gives more chance for the bargaining system to Pareto dominate the leader-follower system.

The second part of the above theorem indicates that, the Pareto range does not shrink to an empty set as $n$ goes to infinity.

### 3.3. Channel efficiencies comparison

It is clear that the channel profit in the coordinated system is obtained when $\lambda_2 = \cdots = \lambda_n = 0$, i.e.,

$$\Pi^C = \frac{(a + bc)^{d+1}}{b} \cdot \frac{-1}{d + 1} \left( \frac{d}{d + 1} \right)^d.$$

It is easy to see, from (4), that the channel efficiency under the leader-follower system is given by

$$\frac{\Pi^L}{\Pi^C} = -(d + 1) \left( \left( \frac{d}{d + 1} \right)^n - 1 \right) \left( \frac{d}{d + 1} \right)^{nd - d},$$

which approaches 0 as $n \to \infty$.

On the other hand, according to (2), we have that the channel efficiency under the bargaining system is

$$\frac{\Pi^B}{\Pi^C} = \left( d + 1 - d \prod_{i=2}^n \left( 1 - \frac{\lambda_i}{d + 1} \right) \right) \prod_{i=2}^n \left( 1 - \frac{\lambda_i}{d + 1} \right)^d.$$

(6)

Obviously, if $\lambda_i = 0$ for $i \in \{2, \ldots, n\}$, the channel efficiency under the bargaining system reaches 100%. As we have discussed previously, this high
efficiency is not implementable because it is achieved in the expense of all the upstream members. Thus, the channel efficiency under the bargaining system we are interested in should satisfy the condition under which the bargaining system Pareto dominates the leader-follower system (i.e., \( \{\lambda_2,\ldots,\lambda_n\} \) satisfies (5)).

Define the maximum channel efficiency under the bargaining system as
\[
\eta^B_n = \max \{ \Pi^B/\Pi^C | \Pi^B_k \geq \Pi^L_k, k = 1,2,\ldots,n \},
\]
which is characterized by the following theorem.

**Theorem 3.** For \( n \geq 3 \), let \( z_0(n) \) be the solution of the following equation other than \([d/(d+1)]^{n-1}:\)
\[
0 = z_0(n)^{d+1} - z_0(n)^d + \left( \frac{d}{d+1} \right)^{d(n-1)} - \left( \frac{d}{d+1} \right)^{d+1(n-1)}.
\]

Then, we have
\[
\eta^B_n = (d+1-dz_0(n))z_0(n)^d \to 100\% \text{ as } n \to \infty.
\]

**Remark:** If \( d = 1 \), then \( z_0(n) = 1 - 2^{1-n} \), hence \( \eta^B_n = 1 - 4^{1-n} \) which converges to 1 exponentially.

From Theorem 3, although the coordination in the bargaining system is not achievable, the channel efficiency under the bargaining system converges to 100% as the number of stages increases to infinity. When we examine the power structure for the channel efficiency to achieve the maximum, we can obtain that \( \lambda_2^* < \cdots < \lambda_n^* \). (The detailed expressions of \( (\lambda_2^*,\cdots,\lambda_n^*) \) are shown in the proof of Theorem 3 in Appendix C.) That is, the power \( 1 - \lambda_{k+1}^* \) of \( P_k \) relative to \( P_{k+1} \) is decreasing in \( k \) (which reflects that the power structure is like a pyramid shape).
4. Extensions

In this section, we study the case of another commonly used demand function, \( D(p_i) = ab^{-p_i} \) \((a > 0, b > 1)\) (exponential demand function, see, e.g., [8]), and show that the main results of Section 3 still hold.

**Theorem 4.** For \( i = 1, 2, \ldots, n \), given \( p_{i+1} \), the bargaining over \( p_i \) results in
\[
 p_i^*(p_{i+1}) = p_{i+1} + \frac{\ln b}{\lambda_i}.
\]

Similar to that in Theorem 1, we can see that the bargaining result of \( p_i \) depends linearly on the relative bargaining power of \( P_i \) to \( P_{i-1} \). Furthermore, the stronger this relative power \( \lambda_i \), the higher the wholesale price \( p_i \).

Since \( p_{n+1} = c \) is given, hence
\[
p_B^k = c + \sum_{i=1}^{n} \frac{\lambda_i}{\ln b},
\]
and the profits of each member and the channel are
\[
\Pi_B^k = ab^c e^{-\sum_{i=1}^{n} \lambda_i} \cdot \frac{\lambda_k}{\ln b},
\]
\[
\Pi_B = ab^c e^{-\sum_{i=1}^{n} \lambda_i} \cdot \sum_{i=1}^{n} \frac{\lambda_i}{\ln b}.
\]

Taking the derivative with respect to \( \lambda_i \) \((i = 2, \ldots, n)\) in (7), we find \( \partial \Pi_B / \partial \lambda_i \leq 0 \), \( \partial \Pi_B^k / \partial \lambda_k \geq 0 \) and \( \partial \Pi_B / \partial \lambda_i \leq 0 \) \((k = 2, \ldots, n, k \neq i)\), which is the same as in Section 3.

By letting \( \lambda_2 = \cdots = \lambda_n = 1 \), we can obtain the results of the leader-follower system as follows
\[
\Pi_L^k = ab^c e^{-n} \cdot \frac{1}{\ln b},
\]
\[
\Pi_L = ab^c e^{-n} \cdot \frac{n}{\ln b}.
\]

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Since $\partial \Pi^B / \partial \lambda_i \leq 0$, we know that the channel profit of the bargaining system is always higher than that of the leader-follower system. Comparing the bargaining system and the leader-follower system, we can find the necessary and sufficient condition under which the bargaining system Pareto dominates the leader-follower system. That is,

$$
\lambda_k \cdot e^{n-\sum_{i=1}^{n} \lambda_i} \geq 1 \quad \text{for} \quad k = 2, \ldots, n.
$$

For the two-stage channel, we have the following theorem, which is the same as Theorem 2.

**Theorem 5.** For $n = 2$, we have $\Pi^L_2 \geq \Pi^B_2$ and $\Pi^L_1 \leq \Pi^B_1$.

For $n \geq 3$, there exist $0 \leq \alpha < \beta \leq 1$ independent of $n$, such that if $\lambda_i \in [\alpha, \beta]$ for all $i = 2, \ldots, n$, then (8) holds.

By letting $\lambda_2 = \cdots = \lambda_n = 0$, the coordinated system’s profit is obtained with

$$
\Pi^C = ab^{-c}e^{-1} \cdot \frac{1}{\ln b}.
$$

Comparing the leader-follower system and the coordinated system, we see that

$$
\frac{\Pi^L}{\Pi^C} = ne^{1-n} \to 0 \quad \text{as} \quad n \to \infty.
$$

But if we consider the maximum channel efficiency of the bargaining system, i.e., $\eta^B_n = \max\{\Pi^B / \Pi^C | \Pi^B_k \geq \Pi^L_k, k = 1, 2, \ldots, n\}$, we have the following theorem.

**Theorem 6.** For $n \geq 3$, let $z_0(n) \in (0, 1)$ be the solution of the following equation:

$$
0 = e^{n-1-z_0(n)}z_0(n) - (n - 1).
$$
Then we have

$$\eta_B^n = \frac{(1 + z_0(n))}{e^{z_0(n)}} \rightarrow 100\% \text{ as } n \rightarrow \infty.$$ 

Remark: Different from Section 3, if we check the best power structure in the proof of Theorem 6, we can see that \(\lambda_2^* = \cdots = \lambda_n^* < 1/n\). This means that in the best power structure, the relative bargaining power is fixed at a constant (depending on \(n\) only) other than a pyramid shape as in Section 3.

5. Concluding remarks

In this paper, we study a multi-stage channel facing a price-sensitive demand with sequential bargaining over the wholesale prices. Our analyses show that powerful downstream members always promote channel coordination while powerful upstream members always increase the effect of double marginalization. In the leader-follower system, the channel efficiency converges to 0% as the number of stages increases to infinity. However, in the bargaining system, under the best power structure, the bargaining system Pareto dominates the leader-follower system, and the channel efficiency converges to 100% as the number of stages increases to infinity.

It is possible to consider other contracts different from the wholesale price contract. However, if such a contract can coordinate the channel with full flexibility (i.e., it can arbitrarily allocate channel profit), then the whole channel is coordinated. For such contracts (e.g., profit sharing, two-part tariff), supplier \(P_n\) can fully extract all channel profit in the leader-follower system, while the profit share of a member is proportional to his bargaining power in the bargaining system (which is similar to that in [14]).
Our paper opens up some possibilities for future researches. For example, in our model, we assume that the demand is deterministic. Models with more general demand functions, or with demand uncertainty, may be considered. Also, a fruitful avenue would be to incorporate information asymmetry into the model by applying the concept of the generalized Nash solution with incomplete information [9].

Acknowledgments

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Appendix A. Proof of Theorem 1

The profit of $P_1$ is $\Pi_1^B = (p_1 - p_2) \cdot (a + bp_1)^d$, and

$$\frac{\partial \Pi_1^B}{\partial p_1} = (a + b(1 + d)p_1 - bdp_2) \cdot (a + bp_1)^{d-1}.$$ 

Hence we know that $\partial \Pi_1^B / \partial p_1 = 0$ has only one feasible solution which maximizes $\Pi_1^B$. In fact, if $d \leq 1$ then $\partial \Pi_1^B / \partial p_1 = 0$ has only one solution; if $d > 1$ then $\partial \Pi_1^B / \partial p_1 = 0$ has two solutions, however, it is easy to see that the solution $p_1 = -a/b$ can not be the maximizer. Further, $\Pi_1^B$ is not maximized when $p_1$ approaches its boundary. Therefore $P_1$ will choose the following $p_1^*(p_2)$ (which is equivalent to the first order condition):

$$a + bp_1^*(p_2) = \frac{d}{d+1} (a + bp_2) = \left(1 - \frac{\lambda_1}{d+1}\right) (a + bp_2).$$

Now, assume that the theorem holds for $i \in \{1, 2, \ldots, k-1\}$, then we consider $i = k$. Given $p_k$, we know from the assumption of induction that
the Nash solution of $p_1$ results in

$$p_1^* \circ \ldots \circ p_{k-1}^*(p_k) = \left(\frac{a}{b} + p_k\right) \prod_{i=1}^{k-1} \left(1 - \frac{\lambda_i}{d+1}\right) - \frac{a}{b}.$$ 

Therefore the profits of $P_k$ and $P_{k-1}$ are

$$\Pi_k^B(p_k) = (a + bp_k)^d(p_k - p_{k+1}) \prod_{i=1}^{k-1} \left(1 - \frac{\lambda_i}{d+1}\right)^d,$$

$$\Pi_{k-1}^B(p_k) = (a + bp_k)^d(p_k^* - p_k) \prod_{i=1}^{k-1} \left(1 - \frac{\lambda_i}{d+1}\right)^d$$

$$= (a + bp_k)^{d+1} - \lambda_{k-1} \frac{b}{d+1} \prod_{i=1}^{k-1} \left(1 - \frac{\lambda_i}{d+1}\right)^d.$$ 

The Nash solution of $p_k$ results in solving:

$$p_k^*(p_{k+1}) = \arg \max_{p_k} (\Pi_k^B)^{\lambda_k} \cdot (\Pi_{k-1}^B)^{1-\lambda_k}$$

$$= \arg \max_{p_k} (a + bp_k)^{d+1-\lambda_k} (p_k - p_{k+1})^{\lambda_k}.$$ 

Hence we obtain a unique maximizer $p_k^*(p_{k+1})$ by checking that the first order condition has only one feasible solution (similar to $k = 1$):

$$a + bp_k^*(p_{k+1}) = \left(1 - \frac{\lambda_k}{d+1}\right) (a + bp_{k+1}).$$ 

By induction, Theorem 1 holds.\[\square\]

**Appendix B. Proof of Theorem 2**

If $n = 2$, since $\lambda_1 = 1$ and $0 \leq \lambda_2 \leq 1$, we have

$$\Pi_2^B = \left(\frac{a + bc}{b}\right)^{d+1} - \lambda_2 \left(1 - \frac{\lambda_1}{d+1}\right)^d \left(1 - \frac{\lambda_2}{d+1}\right)^d$$

$$\leq \left(\frac{a + bc}{b}\right)^{d+1} - 1 \left(\frac{d}{d+1}\right)^{2d} = \Pi_2^B,$$
and
\[
\Pi^B_1 = \frac{(a + bc)^{d+1}}{b} \left( \frac{1 - \lambda_1}{d + 1} \right)^d \left( \frac{1 - \lambda_2}{d + 1} \right)^{d+1} \]
\[
\geq \frac{(a + bc)^{d+1}}{b} \left( \frac{d}{d + 1} \right)^{2d+1} \Pi^L_1.
\]

If \( n \geq 3 \), let \( h_1(x) = x(1 + (1 - x)/d)^{2d} \) and \( h_2(x) = x(1 + (1 - x)/d)^{2d+1} \).

Note that \( h_1(1) = h_2(1) = 1, h'_1(1) = -1 \) and \( h'_2(1) = -1 - d^{-1} < 0 \), hence there exists \( 0 < \mu < 1 \) such that \( h_1(\mu) > 1 \) and \( h_2(\mu) > 1 \). Therefore, there exists \( 0 < \alpha < \mu < \beta < 1 \) (that are independent of \( n \)) such that

\[
1 \leq \alpha \left( 1 + \frac{1 - \alpha}{d} \right)^d \left( 1 + \frac{1 - \beta}{d} \right)^{d+1},
\]
\[
1 \leq \alpha \left( 1 + \frac{1 - \alpha}{d} \right)^d \left( 1 + \frac{1 - \beta}{d} \right)^d.
\]

In the following we will prove the following statement: when \( \lambda_i \in [\alpha, \beta] \) for all \( i = 2, \ldots, n \), then (5) holds.

If \( d > 0 \), then \( 1 + (1 - \beta)/d > 1 \); if \( d < -1 \), then \( 0 < 1 + (1 - \beta)/d < 1 \).

Hence, according to (B.1), we have

\[
1 \leq \alpha \left( 1 + \frac{1 - \alpha}{d} \right)^d \left( 1 + \frac{1 - \beta}{d} \right)^{(n-2)(d+1)},
\]
\[
1 \leq \alpha \left( 1 + \frac{1 - \alpha}{d} \right)^d \left( 1 + \frac{1 - \beta}{d} \right)^{(n-2)d}.
\]

Note that for \( k = 2, \ldots, n \), we have \((n-2)d \leq n(d+1) - 2d - k \leq (n-2)(d+1)\).

Hence for \( k = 2, \ldots, n \), we have

\[
1 \leq \alpha \left( 1 + \frac{1 - \alpha}{d} \right)^d \left( 1 + \frac{1 - \beta}{d} \right)^{n(d+1) - 2d - k}.
\]
Now we are able to show that (5) holds for \( k = 2, \ldots, n \):

\[
\lambda_k \cdot \prod_{i=1}^{n} \left(1 + \frac{1 - \lambda_i}{d}\right)^d \cdot \prod_{j=k+1}^{n} \left(1 + \frac{1 - \lambda_j}{d}\right) \\
= \lambda_k \left(1 + \frac{1 - \lambda_k}{d}\right)^d \cdot \prod_{i=1}^{k-1} \left(1 + \frac{1 - \lambda_i}{d}\right)^d \cdot \prod_{j=k+1}^{n} \left(1 + \frac{1 - \lambda_j}{d}\right)^{d+1} \\
\geq \alpha \left(1 + \frac{1 - \alpha}{d}\right)^d \cdot \prod_{i=1}^{k-1} \left(1 + \frac{1 - \beta}{d}\right)^d \cdot \prod_{j=k+1}^{n} \left(1 + \frac{1 - \beta}{d}\right)^{d+1} \\
= \alpha \left(1 + \frac{1 - \alpha}{d}\right)^d \left(1 + \frac{1 - \beta}{d}\right)^{n(d+1) - 2d - k} \geq 1.
\]

In the above equation, three monotonic functions on \([0, 1]\) are involved:

\[
\frac{\partial}{\partial x} \left\{ x \left(1 + \frac{1 - x}{d}\right)^d \right\} = (1 - x) \frac{1 + d}{d} \left(1 + \frac{1 - x}{d}\right)^{d-1} \geq 0,
\]

\[
\frac{\partial}{\partial x} \left\{ \left(1 + \frac{1 - x}{d}\right)^d \right\} = -x \left(1 + \frac{1 - x}{d}\right)^{d-1} \leq 0,
\]

\[
\frac{\partial}{\partial x} \left\{ \left(1 + \frac{1 - x}{d}\right)^{d+1} \right\} = -x \frac{1 + d}{d} \left(1 + \frac{1 - x}{d}\right)^d \leq 0.
\]

For \( k = 1 \), we have (note that \( \lambda_1 = 1 \))

\[
\lambda_1 \cdot \prod_{i=1}^{n} \left(1 + \frac{1 - \lambda_i}{d}\right)^d \cdot \prod_{j=2}^{n} \left(1 + \frac{1 - \lambda_j}{d}\right) = \prod_{j=2}^{n} \left(1 + \frac{1 - \lambda_j}{d}\right)^{d+1} \geq 1.
\]

In summary, we have proved that (5) holds for \( k = 1, \ldots, n \).

\( \square \)

**Appendix C. Proof of Theorem 3**

To prove this theorem we consider the two cases \( d > 0 \) and \( d < -1 \) separately.
(I) When \( d > 0, \ b < 0 \) and \( a + bc > 0 \).

Let \( x_k = \prod_{i=k}^n (1 + (1 - \lambda_i)/d) \) and \( y_k = x_k x_2^d \), (5) is equivalent to

\[
1 \leq (d + 1) y_{k+1} - dy_k.
\]

Hence we have

\[
y_2 - 1 \leq \left( 1 + \frac{1}{d} \right) (y_3 - 1) \leq \cdots \leq \left( 1 + \frac{1}{d} \right)^{n-1} (y_{n+1} - 1). \tag{C.1}
\]

Since \( y_2 = x_2^{d+1} \) and \( y_{n+1} = x_2^d \), we have

\[
x_2^{d+1} - 1 \leq \left( \frac{d+1}{d} \right)^{n-1} (x_2^d - 1).
\]

Let \( z_2 = x_2 (d/(d+1))^{n-1} \), then

\[
z_2^{d+1} - \left( \frac{d}{d+1} \right)^{(d+1)(n-1)} \leq z_2^d - \left( \frac{d}{d+1} \right)^{d(n-1)}.
\]

Define \( f(t) = t^{d+1} - t^d + (d/(d+1))^{d(n-1)} - (d/(d+1))^{(d+1)(n-1)} \), then \( f(z_2) \leq 0 \). Note that \( f(0) = f(1) > 0 \), \( f((d/(d+1))^{n-1}) = 0 \) and \( f(d/(d+1)) < 0 \), hence there exists a \( z_0(n) \in (d/(d+1), 1) \) such that \( f(z_0(n)) = 0 \).

Further, we have \( f'(t) = t^{d-1}((d+1)t - d) \), hence \( z_0(n) \) is unique and \( z_2 \leq z_0(n) < 1 \). According to (6) we have

\[
\frac{\Pi B}{\Pi C} = (d + 1 - dz_2) z_2^d \leq (d + 1 - dz_0(n)) z_0(n)^d (< 1).
\]

In the following, we need to find the values of \( \lambda_k \ (k = 2, \cdots, n) \) such that the above inequality becomes equal. To do this, we only need all the inequalities in (C.1) become equal and \( z_2 = z_0(n) \). Hence we have

\[
y_2 = x_2^{d+1} = z_2^{d+1} \left( \frac{d+1}{d} \right)^{(n-1)(d+1)} = z_0(n)^{d+1} \left( \frac{d+1}{d} \right)^{(n-1)(d+1)} ,
\]

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and
\[ y_k = (y_2 - 1) \left( \frac{d}{d+1} \right)^{k-2} + 1 = \left( \frac{d+1}{d} \right)^{(d+1)(n-1)-k+2} z_0(n)^{d+1} - \left( \frac{d}{d+1} \right)^{k-2} + 1. \]

Note that
\[ \frac{y_k}{y_{k+1}} = \frac{x_k}{x_{k+1}} = 1 + \frac{1 - \lambda_k}{d}. \]

Thus we know that, if for \( k = 2, \cdots, n \), we choose
\[ \lambda_k^* = 1 - \frac{d}{\left( \frac{d+1}{d} \right)^{(d+1)(n-1)-k+2} z_0(n)^{d+1} - \left( \frac{d}{d+1} \right)^{k-1} + 1}, \]
then \( z_2 = z_0(n) \) and \( \eta_n^B = (d + 1 - dz_0(n))z_0(n)^d \). Also note that when \( n \to \infty \) we have 0 = \( f(z_0(n)) \to z_0(n)^{d+1} - z_0(n)^d \), therefore \( z_0(n) \to 1 \), hence \( \eta_n^B \to 1 \). It remains to show that \( \lambda_k^* \in [0, 1] \), we will prove it together with the next case at the end of the proof.

**II** When \( d < -1, a \geq 0 \) and \( b > 0 \).

This case is very similar to the first case, but some directions of the inequalities are changed.

Let \( x_k = \prod_{i=k}^n (1 + (1 - \lambda_i)/d) \) and \( y_k = x_k x_2^d \), (5) is equivalent to
\[ 1 \leq (d + 1)y_{k+1} - dy_k. \]

Hence we have
\[ y_2 - 1 \geq \left( 1 + \frac{1}{d} \right)(y_3 - 1) \geq \cdots \geq \left( 1 + \frac{1}{d} \right)^{n-1} (y_{n+1} - 1). \]

Since \( y_2 = x_2^{d+1} \) and \( y_{n+1} = x_2^d \), we have
\[ x_2^{d+1} - 1 \geq \left( \frac{d+1}{d} \right)^{n-1} (x_2^d - 1). \]
Let $z_2 = x_2(d/(d+1))^{n-1}$, then
\[
z_2^{d+1} - \left(\frac{d}{d+1}\right)^{(d+1)(n-1)} \geq z_2^d - \left(\frac{d}{d+1}\right)^{d(n-1)}.
\]

Define $f(t) = t^{d+1} - t^d + (d/(d+1))^{d(n-1)} - (d/(d+1))^{d+1(n-1)}$, then $f(z_2) \geq 0$. Note that $f(\infty) = f(1) < 0$, $f((d/(d+1))^{n-1}) = 0$ and $f(d/(d+1)) > 0$, hence there exists a $z_0(n) \in (1, d/(d+1))$ such that $f(z_0(n)) = 0$.

Further, we have $f'(t) = t^{d-1}((d+1)t - d)$, hence $z_0(n)$ is unique and $1 < z_0(n) \leq z_2$. According to (6) we have
\[
\frac{\Pi^B}{\Pi^C} = (d + 1 - dz_2)z_2^d \leq (d + 1 - dz_0(n))z_0(n)^d < 1.
\]

Similar to that in the case (I), if for $k = 2, \cdots, n$, we choose
\[
\lambda^*_k = 1 - \left(\frac{d+1}{d}\right)^{(d+1)(n-1)-k+2}z_0(n)^d - \left(\frac{d}{d+1}\right)^{k-2} + 1
\]
then we can check that $z_2 = z_0(n)$, hence $\eta^B_n = (d + 1 - dz_0(n))z_0(n)^d$. Also note that when $n \to \infty$ we have $0 = f(z_0(n)) \to z_0(n)^d + z_0(n)^d$, therefore $z_0(n) \to 1$, hence $\eta^B_n \to 1$.

Finally, we need to show $\lambda^*_k \in [0, 1]$ for both of the cases. Define
\[
\gamma_n = \left[\left(\frac{d+1}{d}\right)^{(d+1)(n-1)} - \left(\frac{d}{d+1}\right)^{d+1} - 1\right] \left(\frac{d}{d+1}\right)^{-k-2},
\]
then
\[
\lambda^*_k = \frac{d + 1}{d + 1 + d\gamma_n}.
\]

For the case of $d > 0$, we have $z_0(n) \in (d/(d+1), 1)$, thus
\[
d \left(\frac{d+1}{d}\right)^{(d+1)(n-1)} - \left(\frac{d}{d+1}\right)^{d+1} - 1 \geq d \left(\frac{d+1}{d}\right)^{(d+1)(n-2)} - 1 \geq 0.
\]
This means \( \lambda_k^* \in [0, 1] \).

On the other hand, for the case of \( d < -1 \), we have \( z_0(n) \in (1, d/(d+1)) \), thus
\[
d \left[ \left( \frac{d+1}{d} \right)^{(d+1)(n-1)} z_0(n)^{d+1} - 1 \right]
\leq d \left[ \left( \frac{d+1}{d} \right)^{(d+1)(n-2)} - 1 \right] \leq 0.
\]
This also means \( \lambda_k^* \in [0, 1] \).

Therefore, the proof of this theorem is completed. \( \square \)

**Appendix D. Proof of Theorem 4**

The profit of \( P_1 \) is \( \Pi_1^B = ab^{-p_1} \cdot (p_1 - p_2) \). Hence
\[
p_1^*(p_2) = p_2 + \frac{\lambda_1}{\ln b} = p_2 + \frac{1}{\ln b}.
\]

Now, assume that the theorem holds for \( i \in \{1, 2, \ldots, k-1\} \), then we consider \( i = k \). Given \( p_k \), we know from the assumption of induction that the Nash solution of \( p_1 \) results in
\[
p_1^* \circ \ldots \circ p_{k-1}^*(p_k) = p_k + \frac{\sum_{i=1}^{k-1} \lambda_i}{\ln b}.
\]
Therefore the profits of \( P_k \) and \( P_{k-1} \) are
\[
\Pi_k^B(p_k) = ab^{p_k} e^{\sum_{i=1}^{k-1} \lambda_i (p_k - p_{k+1})},
\]
\[
\Pi_{k-1}^B(p_k) = ab^{p_k} e^{\sum_{i=1}^{k-1} \lambda_i (p_{k-1}^* - p_k)}.
\]
The Nash solution of \( p_k \) results in solving:
\[
p_k^*(p_{k+1}) = \arg \max_{p_k} (\Pi_k^B)^{\lambda_k} \cdot (\Pi_{k-1}^B)^{1-\lambda_k}.
\]

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Hence we obtain a unique maximizer $p^*_k(p_{k+1})$, where

$$p^*_k(p_{k+1}) = p_{k+1} + \frac{\lambda_k}{\ln b}.$$ 

By induction, Theorem 4 holds. □

**Appendix E. Proof of Theorem 5**

If $n = 2$, since $\lambda_1 = 1$ and $0 \leq \lambda_2 \leq 1$, we have

$$\Pi^B_2 = ab^{-c}e^{-(\lambda_1 + \lambda_2)} \frac{\lambda_2}{\ln b} \leq ab^{-c}e^{-2} \frac{1}{\ln b} = \Pi^B_1,$$

and

$$\Pi^B_1 = ab^{-c}e^{-(\lambda_1 + \lambda_2)} \frac{\lambda_1}{\ln b} \geq ab^{-c}e^{-2} \frac{1}{\ln b} = \Pi^B_1.$$ 

If $n \geq 3$, assume $0 < \alpha < \beta < 1$ such that, when $\lambda_i \in [\alpha, \beta]$ for all $i = 2, \ldots, n$, then (8) holds. Hence we must have (by (8) and $\lambda_1 = 1$)

$$1 \leq \alpha e^{1-\alpha}e^{(n-2)(1-\beta)}.$$ 

Note that, whenever the above inequality holds for $n = 3$, it holds for all $n \geq 3$, hence we must have

$$1 \leq \alpha e^{1-\alpha}e^{1-\beta}. \quad (E.1)$$

Define $h(x) = xe^{2-2x}$, we have $h(1) = 1$ and $h'(1) = -1$. Hence there is a $0 < \mu < 1$ such that $h(\mu) > 1$. Therefore, there exists $0 < \alpha < \mu < \beta < 1$ such that (E.1) holds.

□

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Appendix F. Proof of Theorem 6

Let \( x_k = \sum_{i=k}^{n} \lambda_{i} \). Then (8) is equivalent to

\[
1 \leq (x_k - x_{k+1})e^{n-1-x_2}.
\]

Taking the sum from \( k = 2 \) to \( n \), we have

\[
n - 1 \leq x_2 e^{n-1-x_2}.
\]

Define \( f(t) = te^{n-1-t} - (n-1) \), then \( f(x_2) \geq 0 \). Note that \( f(0) = -(n-1) < 0 \) and \( f(1) = e^{n-2} - (n-1) > 0 \), hence there exists a \( z_0(n) \in (0,1) \) such that \( f(z_0(n)) = 0 \). Further, we have \( f'(t) = (1-t)e^{n-1-t} \), hence \( z_0(n) \) is unique and \( 0 < z_0(n) \leq x_2 \). According to (7) and (9) we have

\[
\frac{\Pi_B^C}{\Pi_C} = (1 + x_2)e^{-x_2} \leq (1 + z_0(n))e^{-z_0(n)} < 1.
\]

If for \( k = 2, \ldots, n \), we choose

\[
\lambda^*_k = e^{z_0(n)+1-n} = \frac{z_0(n)}{n-1} \in [0,1],
\]

then we can check that \( x_2 = z_0(n) \), hence \( \eta_n^B = (1 + z_0(n))e^{-z_0(n)} \). Also note that when \( n \to \infty \) we have \( z_0(n) = (n - 1)e^{-1-n} \cdot e^{z_0(n)} \to 0 \), hence \( \eta_n^B \to 1. \)

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