Web System Upgrading with Transaction Failure and Strategic Customers

Abstract:
This paper considers pricing and web system upgrading problems for an online retailer facing a group of strategic customers. Due to various website issues, there is possibility of transaction failure in the process of customer online purchasing. Strategic customers will anticipate this possibility and make purchasing decisions based on their belief on transaction success probability (TSP). First, we prescribe a threshold policy for customer purchasing: the customer will buy the product if his valuation for this product is above a threshold, and will not otherwise. The threshold increases as TSP decreases, customer transaction cost increases, or customers become more risk averse. Second, we derive the optimal price of each period and identify the optimal policy for web system upgrading: there exists a threshold for each period such that the online retailer should upgrade their web system to the state of art (i.e., achieve highest available TSP) if current TSP is below the threshold, and should not upgrade otherwise. The threshold (total discounted profit) increases as customer transaction cost decreases, customer valuations for the product become higher, or customers become more (less) risk averse. Third, we find that the online retailer tends to price higher if it ignores customer strategic behavior. The cost of ignoring customer strategic behavior is substantial. The profit loss rate of ignoring customer strategic behavior increases as customer transaction cost increases, customer valuations for the product become lower, or customers become more risk averse.

Index Terms: online retailing; transaction failure; technology adoption; strategic customers

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I. INTRODUCTION

As information technology develops, more and more retailers open their online channels for sales, thus online sales see dramatic increases in the past several years. For example, according to Spending Pulse, a MasterCard Advisors report, there is a more than 15% increase of online sales in 2010 over 2009 [6]. An increasing number of consumers conduct their shopping through Internet. A PwC survey shows that in 2011 almost 92% of those surveyed were buying on the Web, while in 2009 they accounted for only 80% [34].

While consumers are substantially more and more likely to conduct a shopping online, they are increasingly unwilling to tolerate low levels of service online [31]. Website problems during the process of online shopping are ubiquitous. According to a poll held by Tealeaf, a leader in online customer experience management software, 88% of the people polled reported online problems during transactions. Hundreds of respondents submitted vignettes of bad online experiences, most concerning failed, inaccurate or incomplete transactions, “endless loops” in a business process or inability to simply checkout at the websites of online retailers [31,32].

Website issues often make consumers abandon transactions, leading to transaction failures. Moreover, website issues result in negative customer experiences. The customers may tell their friends about their bad experiences, or write them down on some related forums. Other online shoppers can have easy access to the comments, which leads to word-of-mouth effects that damage the reputations of online retailers.

By Consumer Mercantile Model in [23], consumer purchasing activity consists of three phases: pre-purchase interaction, purchase consummation and post-purchase interaction. Consumers obtain various types of information on online retailers in the phase of pre-purchase. It is documented that these kinds of information do affect consumers’ attitudes toward a website (or an online retailer). The attitudes, in turn, lead to their intentions to use the website and the eventual acceptance of the website (or the online retailer) [14]. One type of information that affect the customers’ final choice is the
website’s (the online retailer’s) ability to complete transactions without any problem [31]. Therefore, website issues can lead to transaction failure and customer defection, causing huge losses to online retailers. It is reported that retailers who operate in the online channel may have lost more than $44 billion dollars over this past year as a result of transaction problems on their website [5]. This fact gives rise to an important question: how to improve online retailers’ profits by reducing the losses caused by website issues?

This paper takes an initial step to answer this question. We propose adjusting selling prices and upgrading web systems to reduce online retailers’ profit losses from website issues. The network operation and maintenance engineers of Baidu Inc. (China's largest search engine service provider) explain to the authors that website issues are mainly attributed to hardware and software problems of web systems. The hardware problems mainly include the lack of enough severs to support huge traffic on the website and the unstable network supported by Internet Service Providers. On the other hand, the software problems are mainly caused by redundant processing logic which results in denials of access, and unreasonable web design which brings inconvenience or failures to online customers. Therefore online retailers can upgrade the web systems by improving the hardware capacity (e.g. purchasing new severs) and optimizing the processing logic and the web design. Since these actions could be very costly, then how to price and when to upgrade web systems are crucial problems for online retailers.

We propose an analytical model in which an online retailer sells a type of product to a group of strategic customers through Internet. Comparing to nonstrategic customers who ignore the effect of transaction failure, strategic customers anticipate the transaction failure probability and make purchasing decisions based on the utility of a successful purchasing and the disutility of an unsuccessful one. All the customers have unit demand in each period and heterogeneous valuations for the product. The following results are obtained:

First, we characterize a threshold policy for strategic customer purchasing: There
exists a unique threshold such that a customer will buy the product if his valuation is greater than the threshold; and will not buy the product otherwise. We further demonstrate that a customer will be more likely to conduct an online purchasing if the website is technically more reliable (lower probability of website issues), the transaction cost of the purchasing is lower, or the customer is less risk averse.

Second, this paper provides guidelines for the online retailer on how to price and when to upgrade the web system. We propose a multi-period model in which the online retailer has an opportunity to set price and upgrade its web system at the beginning of each period. The optimal price for each period is derived and a threshold policy is proposed for upgrading: there exists a threshold for each period such that the online retailer should upgrade the web system to the highest available TSP if the current TSP is below the threshold, and not upgrade otherwise. Sensitive analysis is conducted to investigate how the threshold and the optimal profit of the online retailer change with various model parameters.

Third, this paper discusses the online retailer’s cost of ignoring customer strategic behavior. It is proved that the online retailer tends to price higher when ignoring customer strategic behavior. Numerical examples show that the profit loss is substantial (sometimes the profit loss rate can be up to 65%). To alleviate the negative effect of ignoring customer strategic behavior, the online retailer should (1) increase customer valuations for the product by better product design or more impressive advertising; (2) decrease the customer transaction cost by providing better navigation aids. Besides, if the customers are less risk averse, the negative effect of ignoring customer strategic behavior is smaller.

Finally, some extensions and variations of the model are examined. It is shown that main findings and managerial insights remain true for these extensions and variations.

The rest of our paper proceeds as follows. Section II reviews the literature related to our paper. Section III presents the basic model and characterizes the customers’ purchasing decisions. In Section IV, the online retailer’s pricing and upgrading decisions
are studied and sensitivity analysis is conducted. In Section V, the cost of ignoring customer strategic behavior is investigated. In Section VI, some extensions and variations of the model are examined. Finally, in Section VII, we come to conclusions and future research directions.

II. LITERATURE REVIEW

This paper is related to four streams of studies: (1) online retailing; (2) customer behavior; (3) technology adoption; (4) risk aversion.

A. Online Retailing

In recent years, online retailing attracts a great deal of attention, and meanwhile tremendous studies emerge to consider issues related to this field. In general, the studies in this field can be divided into two categories. The first focuses on the design of channel structure and the influences of online retailing entry over traditional retailing forms. Examples of this category include mixed channel (e.g., [25]), price competition (e.g., [2]), channel substitution (e.g., [17]) and so on. The second is devoted to designs and attributes of the online retailing website (e.g., [11,14,16]). Among the researches in this category, some focus on characteristics of the website (e.g., perceived risk, trust, service quality, ease of use etc.), and others provide suggestions on how to design the website by exploring consumer characteristics, such as consumer shopping orientations (e.g., convenience oriented, price oriented, experiential oriented etc.), demographic variables (e.g., educational level, age, gender etc.), psychological variables (e.g., attitudes towards online shopping, intention to use Internet for information search, risk aversion etc.). Chang et al. [10] provide a comprehensive review of this literature.

Our paper falls into the second category and contributes to this literature by considering transaction/service failure due to website issues. A number of researches are devoted into transaction/service failure of online retailing (e.g., [1,21,36]). These researches mainly focus on the causes and costs of transaction/service failure, and the
value of service recovery. Comparing to all these empirical researches, our paper proposes a mathematical model to provide guidelines for online retailers on pricing decisions and technology upgrading policy. More importantly, we introduce the notion of strategic customers who anticipate the possibility of transaction failure and make purchasing decisions based on their beliefs on transaction success probability which is not considered in those researches on transaction/service failure.

B. Customer Behavior

In traditional operations management (OM) literature, customer demand is often assumed to be exogenous, i.e., demand functions are usually set as specified functions of price or other product attributes [29]. However, in the real world, all customers do, at some point, actively evaluate alternatives and make choices, e.g., how much to pay, which product to buy, when to buy, etc. [35]. That is to say, the customer will engage in decision-making processes and are not simply governed by the demand profile specified at the outset. It is shown that these customers’ decision processes (customer behavior) deserve attention and for many practical problems, neglecting these decision processes on the demand side may have significant repercussions [35].

A few studies discuss the firms’ decision incorporating strategic customer behavior. Su [37] develops a model of dynamic pricing in the presence of strategic consumers who decide the timing of purchasing. Su [39] considers a monopolist firm selling a fixed capacity with speculators and strategic consumers. Consumers may strategically choose when to purchase, and they may also choose to purchase from the firm or from the speculators. Su [38] studies a dynamic pricing problem for a class of products with stable consumption patterns (e.g., household items, staple foods). Consumers may stock up the product at current prices for future consumption, but they incur inventory holding costs. Su and Zhang [41] consider a model with strategic customers who anticipate the likelihood of stockouts and determine whether to visit the seller. Yin et al. [43] compare two inventory display formats: Display All (DA) and Display One (DO) for a retailer who
sells a limited inventory of a product in the presence of strategic consumers who decide the timing of purchasing. Liu and van Ryzin [26,27] propose strategic capacity rationing to cope with customer strategic behavior of waiting for price markdown. Su and Zhang [40] study the impact of strategic customer behavior on supply chain performance. Customers anticipate future price markdown and choose purchase timing to maximize their expected surplus.

Some researches consider customer irrational behaviors. For example, Broder and Rusmevichientong [4] consider a dynamic pricing model in which a monopolist prices a product to a group of customers, who independently make purchasing decisions based on the price offered according to a logit choice model. Veeraraghavan and Debo [42] study consumers’ herd behaviors in queues. For more literature on consumer behavior, please refer to [35] and [29].

Our paper contributes to this literature by considering strategic customers who anticipate the probability of online transaction failure and make purchasing decisions based on their beliefs over this probability.

C. Technology Adoption

The studies on technology adoption have a long history (see [20,22] for comprehensive surveys). Schumpeter [30] introduces the concept of “creative destruction”, which means the discovery and adoption of a new innovation effectively destroys the old technology by rendering it obsolete. Balcer and Lippman [3] illustrate that the firm will adopt the current best practice if its technological lag exceeds a certain threshold or even purchase a technology that has been available although it was not profitable to do so in the past. Chambers [8] investigate learning effects of technology; Chambers and Kouvelis [9] study the influence of competition over new technology adoption; Cho and McCardle [12] consider the adoption of multiple dependent technologies. While most of existing literature directly sets profit (or profit rate) as a function of technology level/technology lag, our paper employs a consumer choice model to derive the profit function, which
enables us to study how consumer characteristics (e.g., risk attitude of consumers) affect the firms’ pricing and upgrading decisions. Liu and Ozer [24] also utilize a consumer choice model to analyze a product family management problem. However, their consumer choice model is completely different from ours.

D. Risk Aversion

In the field of OM, the studies incorporating risk aversion are rich (e.g., [13,15,18,19,43]). Choi et al. [13] and Eeckhoudt et al. [15] investigate the optimal decisions of risk averse newsvendors. Xiong et al. [43] consider overbooking problems of risk averse decision makers. Gan et al. [18] provide the definition of coordination of supply chains consisting of risk averse members. Gan et al. [19] propose a new contract to coordinate a supply chain with a risk neutral supplier and a downside-risk-averse retailer. While these researches assume the firms (the decision makers of the firms) to be risk averse, our paper assume that the customers are risk averse. More importantly, the research questions of our paper are totally different from the above ones.

In general, the characterizations of risk averse consist of (1) mean-variance framework (e.g., [13]), (2) concave utility functions according to expected utility theory (e.g., [15,43]), and (3) other measurements like downside-risk, value-at-risk (e.g., [19]). Our paper employs the second one and assumes the customers have an identical utility function that is increasing and concave.

III. MODEL AND CUSTOMER PURCHASING BEHAVIOR

Consider a firm who sells a single product through Internet to a fixed population of online shoppers. The firm produces the product at a unit cost of $c$ and sells it at a price of $p$ ($p>c$). The production cost $c$ is exogenous and normalized to 0 for simplicity. We consider a market of $N$ customers. The market size $N$ is deterministic and very large. Customers have unit demand and heterogeneous valuations for the product which is a random valuation drawn from a common cumulative distribution function (CDF) $F(v)$. 
Let $f(v)$ be the corresponding probability density function (PDF) and $F(v) = 1 - F(v)$ be the corresponding complementary cumulative distribution function. Furthermore, we assume $f(v)$ is continuous and $F(v)$ has increasing hazard rate (failure rate), i.e., the hazard rate function $r(v) := f(v) / F(v)$ is a weakly increasing function. Many distribution functions have increasing hazard rate, including uniform, exponential, normal distributions, Weibull distribution with shape parameter $\alpha > 1$, Gamma distribution with shape parameter $\alpha > 1$ etc. The customers are risk averse and have an identical utility function $u(\cdot)$, which is an increasing and concave function with $u(0) = 0$ and $u(\infty) = \infty$. The customers have a homogeneous transaction cost $h (h > 0)$ for the online purchasing, including time costs of searching, bargaining, paying bill by personal Internet bank, etc. [14].

The customers will make unsuccessful purchasing with a probability $1-q$ due to various website issues (e.g., slow speed, web errors). We refer to $q$ as transaction success probability (TSP). We assume that the customers have correct anticipations (observations) on TSP. This assumption is made based on the following facts: Nowadays, more and more people have access to Internet with computers or other communication instruments, which leads more and more online shoppers write down their past experiences of shopping on some websites. Customers can estimate TSP by examining the experiences of former online shoppers.

Next we will characterize the purchasing behavior of a strategic customer with random realization $v$ as valuation for a given selling price $p$. It is easy to know that the customer will not buy the product if his valuation $v < p + h$. If $v \geq p + h$, the customer will decide by comparing the utility of buying the product with that of not buying. In particular, the strategic customer will (not) try to buy the product if

$$qu(v - p - h) + (1-q)u(-h) \geq (<)u(0).$$

(1)
Note that “≥” of Inequality (1) is the condition that a customer will try to buy the product online, but doesn’t necessarily finally buy due to website issues. For simplicity we omit the words “try to” if there is no confusion. The following proposition characterizes the optimal purchasing decisions for customers.

**Proposition 3.1:** Suppose the selling price \( p \) is given. There exists a unique threshold \( x(q,h) > h \) satisfying

\[
qu(x(q,h)−h)+(1−q)u(−h)=0
\]

(2)

such that a customer will buy the product if his valuation \( v \geq p+x(q,h) \); and will not buy the product if \( v < p+x(q,h) \).

Proposition 3.1 prescribes the optimal purchasing decisions for the customers in the presence of various website issues. The threshold \( x(q,h) \) is a function of \( q \) and \( h \) representing an invisible cost of online purchasing behavior which is determined by transaction cost \( h \) and TSP \( q \). A threshold policy is optimal for the customers: if the valuation for the product is above \( x(q,h) + p \), the customer should buy the product and otherwise the customer should not buy. A more interesting question is: How the threshold changes with \( q \) and \( h \)? The following proposition answers the question.

**Proposition 3.2:** \( x(q,h) \) is decreasing with respect to \( q \) and increasing with respect to \( h \).

Proposition 3.2 indicates that customers are more likely to buy the product if TSP \( q \) is higher, or if the transaction cost \( h \) is lower. The latter result is consistent with previous results of [30], which finds that customers are less price-sensitive and more likely to purchase the product when information provided on the website is easier to navigate (i.e., lower transaction cost).

Another question is that how \( x(q,h) \) changes with the customers’ degree of risk aversion. According to [33], an increase in the degree of risk aversion can be represented by an increasing concave transformation. Let \( w(·)=\phi(u(·)) \), where \( \phi(·) \) is an increasing concave function with \( \phi(0) = 0 \). Thus, \( w(·) \) represents a higher degree of
risk aversion than \( u(\cdot) \). Let \( x^u(q,h) \) and \( x^w(q,h) \) stand for the thresholds corresponding to \( u(\cdot) \) and \( w(\cdot) \) respectively. Hereafter we assume that the functions \( u(\cdot), \varphi(\cdot) \) are differentiable.

**Proposition 3.3:** \( x^u(q,h) \leq x^w(q,h) \).

Proposition 3.3 indicates that customers are less likely to by the product if they are more risk averse. For simplicity, hereafter we say \( y \) is increasing (decreasing) with respect to the customers’ degree of risk aversion if \( y^u \geq y^w \) (\( y^w \leq y^u \)), where \( y^w \) and \( y^u \) are two quantities corresponding to \( u(\cdot) \) and \( w(\cdot) \) respectively (note that \( w(\cdot) \) represents higher degree of risk aversion than \( u(\cdot) \)).

Propositions 3.1-3.3 above enable us to derive the firm’s total demand and to discuss how various model parameters, such as \( q, h \) and customers’ degree of risk aversion, affect the firm’s total demand and profit.

**IV. PRICING AND WEB UPGRADING**

Adjusting selling price \( (p) \) and making web upgrading to increase TSP \( (q) \) are effective solutions to solve the transaction failure problem caused by various website issues. Next we study the firm’s joint decisions of pricing and web upgrading. To this end, we consider an \( n \)-period model. We assume that in Period \( i (i=1,2,\ldots,n) \), TSP cannot exceed \( \bar{q}_i \) due to technology limitations and that \( 0 < q = \bar{q}_1 \leq \bar{q}_2 \leq \cdots \leq \bar{q}_n \leq 1 \), which means the highest technology available develops. In Period \( i (i=1,2,\ldots,n) \), the timing of events is as follows.

1. Observing current TSP \( q_i \), the firm decides whether to upgrade its web system to increase TSP and how much to increase by (i.e., choose \( q \in [q_i, \bar{q}_i] \)). The firm incurs a fixed cost \( K_i \) for the system upgrade. That means, the cost for the upgrade
is $K_i \delta(q - q_i)$, where $\delta(x) = 1$ if $x > 0$, and $\delta(x) = 0$, if $x = 0$.

(2) The firm decides the selling price $p$ based on $q$.

(3) Customers make purchasing decisions according to $p$ and $q$. Here we assume the purchasing behaviors of the customers in each period only depend on the price and TSP in current period, irrelevant to those in other periods. Besides, we assume customers can correctly expect the value of $q$. This rational expectation assumption is also adopted in other literature (e.g., [26,41]). We will drop this assumption and consider an adaptive learning model Section VI.

The state transition among periods follows

$$q_i = q^*_i, \ i = 2,3,\ldots,n,$$

where $q_i$ stands for TSP at the beginning of Period $i$ (before upgrading decision), and $q^*_i$ is optimal TSP chosen in Period $i-1$ (after upgrading decision).

We employ the backward induction to solve this problem. In Subsection A, we derive the optimal price in each period for given TSP. In Subsection B, we derive the optimal upgrading policy based on the optimal pricing decision in Subsection A.

A. Pricing Decisions

In this subsection, we derive the optimal price in each period for any given TSP $q$. With selling price $p$, according to Proposition 3.1, only customers with valuation $v \geq p + x(q,h)$ will buy the product. According to the assumption in Section III that the market size $N$ is deterministic and very large, the demand with selling price $p$ is $N(1-F(p+x(q,h))) = NF(p + x(q,h))$. Note that the transactions only succeed with a probability $q$. Then the firm’s expected profit in a single period with TSP $q$ and selling price $p$ is

$$\pi(p,q) = pqNF(p + x(q,h)).$$

(4)

Lemma 4.1: For any given $0 < q < 1$, $\pi(p,q)$ is unimodal with respect to $p$ and the
optimal price $p^*(q)$ is the unique solution of

$$1 - p \cdot r(p + x(q,h)) = 1 - p \cdot \frac{f(p + x(q,h))}{F(p + x(q,h))} = 0; \quad (5)$$

Equipped with Lemma 4.1, we have the following properties for the optimal price and profit in each period.

**Proposition 4.1:**
(a) $p^*(q)$ is increasing with respect to $q$, decreasing with respect to $h$ and customers’ degree of risk aversion.
(b) $\pi^*(q) := \pi(p^*(q), q)$ is increasing with respect to $q$, decreasing with respect to $h$ and customers’ degree of risk aversion.

Proposition 4.1 shows that the online retailer should price higher and will get a higher profit if its website’s ability to complete transactions without any problems is higher, customers’ transaction cost is lower, or if customers are less risk averse.

Next we investigate how the optimal price and profit change with the customer valuations for the product. To this end, we introduce the notion of the hazard rate order for random variables. Given two non-negative random variables $X$ and $Y$ with PDFs $f_X(x)$ and $f_Y(y)$. The corresponding CDFs are $F_X(x)$ and $F_Y(y)$. We define that $X$ is no more than $Y$ according to hazard rate order (denoted by $X \leq_{hr} Y$, or equivalently $F_X(\cdot) \leq_{hr} F_Y(\cdot)$) if

$$r_X(v) := f_X(v) / F_X(v) \geq r_Y(v) := f_Y(v) / F_Y(v)$$

for all $v \geq 0$, or equivalently, $F_X(v) / F_Y(v)$ is decreasing in $v$ over $[0, +\infty)$. Hazard rate order, which often appears in decision theory, is an effective way to compare two nonnegative random variables. For example, if $X$ follows a uniform distribution over $[0, A_X]$ and $Y$ follows a uniform distribution over $[0, A_Y]$, then $X \leq_{hr} Y$ is equivalent to $A_X \leq A_Y$.

**Proposition 4.2:** Suppose that $p^*_{F_X}$ and $\pi^*_{F_X}$ ($p^*_{F_Y}$ and $\pi^*_{F_Y}$) are the optimal price and single-period profit corresponding to CDF $F_X(\cdot)$ (CDF $F_Y(\cdot)$). If both $F_X(\cdot)$ and $F_Y(\cdot)$
Proposition 4.2 indicates that both the optimal price and single-period profit are higher if customer valuations for the product are higher (or the product is more attractive to the customers). For simplicity we say that \( y \) is increasing (decreasing) with respect to customer valuations, if
\[
F_X(q) \leq F_Y(q) \quad \Rightarrow \quad y_X \leq y_Y \quad (y_X \geq y_Y),
\]
where \( y_X \) and \( y_Y \) are two quantities corresponding to CDFs \( F_X(q) \) and \( F_Y(q) \) respectively.

**B. Upgrading Policy**

The web system upgrading problem can be formulated as the following dynamic program:

\[
V_i(q_i) = \max_{q \in [q_i, \bar{q}_i]} \left\{ -K_i \delta(q - q_i) + \pi^*(q) + \beta V_{i+1}(q) \right\}, \quad i = 1, 2, \ldots, n,
\]

where \( V_i(q_i) \) stands for the optimal total discounted profit from Period \( i \) to Period \( n \), with TSP \( q_i \) at the beginning of Period \( i \). At the end of Period \( n \), the terminal value is \( V_{n+1}(q) = 0 \) for any \( q \in [q_n, \bar{q}_n] \). The following proposition provides a start for the optimal upgrading policy for Problem (6).

**Proposition 4.3:** Let \( Q^* = (q_1^*, q_2^*, \ldots, q_n^*) \) be the optimal solution to Problem (6) with TSP \( q_1 \) at the beginning of Period 1. Then \( q_i^* \in [q_{i-1}, \bar{q}_i] \), \( i = 1, 2, \ldots, n \), where \( q_0^* = q_1 \).

Proposition 4.3 states that the firm should either maintain the current TSP without upgrading, or make an upgrade to generate the highest available TSP. This proposition also helps reduce the complexity of Problem (6) significantly. According to Proposition 4.3, we only need to choose \( q_i^* \) from the set \( [q_i, \bar{q}_i] \) in contrast to the interval \([q_i, \bar{q}_i]\). Thus, Problem (6) is equivalent to

\[
V_i(q_i) = \max \{ J_i^*(q_i), J_i^{au}(q_i) \}, \quad i = 1, 2, \ldots, n
\]

where
\[ J_i^u(q_i) \equiv -K_i + \pi^*(\bar{q}_i) + \beta V_{i+1}(\bar{q}_i), \quad (8) \]

\[ J_i^{nu}(q_i) \equiv \pi^*(q_i) + \beta V_{i+1}(q_i). \quad (9) \]

Let \( J_i(q_i) = J_i^u(q_i) - J_i^{nu}(q_i) \) be the net profit from upgrading the web system at the beginning of Period \( i \). Then Problem (7) is equivalent to

\[ V_i(q_i) = \max \left\{ J_i(q_i), 0 \right\} + J_i^{nu}(q_i), \quad i = 1, 2, \ldots, n \quad (10) \]

From Eqn. (10), whether to upgrade the web system depends on the sign of \( J_i(q_i) \): if \( J_i(q_i) > 0 \), it is optimal to upgrade the web system and if \( J_i(q_i) \leq 0 \), it is optimal to maintain the current TSP.

**Proposition 4.4:** For \( i = 1, 2, \ldots, n \),

(a) \( J_i^{nu}(q_i) \) is regardless of \( q_i \);

(b) \( J_i^{nu}(q_i) \) is increasing in \( q_i \);

(c) \( J_i(q_i) \) is decreasing in \( q_i \);

(d) \( V_i(q_i) \) is increasing in \( q_i \);

(e) The optimal policy for the firm is \( q_i^* = q_i \) (i.e., not upgrade) if \( q_i \geq T \), and \( q_i^* = \bar{q}_i \) (i.e., upgrade) otherwise, where

\[ T = \begin{cases} \max \{ q \in [q_i, \bar{q}_i] : J_i(q) \geq 0 \}, & \text{if } \max_{q \in [q_i, \bar{q}_i]} J_i(q) \geq 0, \\ 0, & \text{otherwise}. \end{cases} \quad (11) \]

Part (b) of Proposition 4.4 shows that the profit of maintaining the current web system is larger when current TSP is higher. Part (c) states the net profit of upgrading the web system gets lower if the current TSP becomes higher. Part (d) indicates that the optimal total discounted profit increases with current TSP. Part (e) prescribes the optimal upgrading policy for the firm: there exists a threshold such that the firm should upgrade its web system to achieve the highest available TSP if the current TSP is below
the threshold, and should not upgrade otherwise.

Next we investigate how the firm’s optimal total discounted profit and the upgrading thresholds change with model parameters.

**Proposition 4.5**: \( V_i(q_i) \ (i=1,2,\cdots,n) \) is decreasing with respect to \( K_j \ (i \leq j \leq n) \), \( h \) and customers’ degree of risk aversion, and increasing with respect to customer valuations for the product.

**Proof**: This proposition can be easily proved by induction. We omit the proof here to save space. ■

Proposition 4.5 indicates that the firm’s total discounted profit is higher if upgrading costs are lower, customer valuations for the product are higher, customer transaction cost is lower, or if customers are less risk averse. These results are intuitively understandable.

Next we investigate how the optimal threshold \( T \) changes with model parameters. To this end, we do some numerical examples in which \( F(x) \) is the CDF of uniform distribution over \([0, A]\), \( n = 5 \), \( N = 1000 \), \( u(x) = 1 - \exp(ax) \), \( K_i = 500 \), \( i = 1,2,\cdots,5 \), \( \bar{q}_1 = 0.5 \), \( \bar{q}_2 = 0.625 \), \( \bar{q}_3 = 0.75 \), \( \bar{q}_4 = 0.875 \), \( \bar{q}_5 = 1 \), \( \beta = 0.9 \). First, we set \( h = 4 \) and \( a = 0.01 \), and investigate how threshold \( T \) changes with \( A \). The results are listed in Table I. It demonstrates that \( T \) increases as \( A \) increases, which means the firm is more likely to upgrade its web system when customer valuations for the product are higher. This is because it is more profitable to upgrade the web system when the product is more attractive to the customers (i.e., \( A \) is larger).

Second, we set \( a = 0.01 \) and \( A = 40 \), and investigate how \( T \) changes with \( h \). The results are listed in Table II. We find that \( T \) becomes smaller when customer transaction cost becomes higher. This indicates that the firm is less likely to upgrade its web system when customer transaction cost is higher. It is also very natural because it is less profitable to upgrade the web system when customer transaction cost is higher,
which results in smaller population of customers who try to buy the product.

Third, we set $h = 4$ and $A = 40$, and investigate how $T$ changes with customers’ degree of risk aversion. The results are listed in Table III, in which larger $a$ represents higher degree of risk aversion. The result demonstrates that the firm is more likely to upgrade the web system when facing more risk-averse customers. The explanation is as follows. When customers are more risk averse, they are more sensitive to the possibility of transaction failure. Thus, upgrading the web system, which increases TSP, attracts more customers to buy the product. This means upgrading the web system is more profitable when customers are more risk averse, thus $T$ increases with customers’ degree of risk aversion.

Interestingly, further numerical analysis demonstrates that $T$ has no explicit relationship with period number $i$ ($i = 1, 2, \cdots, 5$). The relationship between threshold $T$ and period number $i$ depends on the interactions of several parameters, such as $\bar{q}_i$ and $K_i$ ($i = 1, 2, \cdots, 5$).

### TABLE I

<table>
<thead>
<tr>
<th>THRESHOLDS V.S. CUSTOMER VALUATIONS FOR THE PRODUCT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ for Period 1 $T$ for Period 2 $T$ for Period 3 $T$ for Period 4 $T$ for Period 5</td>
</tr>
<tr>
<td>------------------------------------------------</td>
</tr>
<tr>
<td>$A=35$</td>
</tr>
<tr>
<td>$A=40$</td>
</tr>
<tr>
<td>$A=45$</td>
</tr>
<tr>
<td>$A=50$</td>
</tr>
</tbody>
</table>

### TABLE II

<table>
<thead>
<tr>
<th>THRESHOLDS V.S. CUSTOMER TRANSACTION COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ for Period 1 $T$ for Period 2 $T$ for Period 3 $T$ for Period 4 $T$ for Period 5</td>
</tr>
<tr>
<td>-----------------------------------------</td>
</tr>
<tr>
<td>$h=2$</td>
</tr>
</tbody>
</table>
TABLE III
THRESHOLDS V.S. CUSTOMERS’ DEGREE OF RISK AVERSION

<table>
<thead>
<tr>
<th></th>
<th>T for Period 1</th>
<th>T for Period 2</th>
<th>T for Period 3</th>
<th>T for Period 4</th>
<th>T for Period 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>a=0.01</td>
<td>0.4483</td>
<td>0.574</td>
<td>0.6994</td>
<td>0.8245</td>
<td>0.9496</td>
</tr>
<tr>
<td>a=0.03</td>
<td>0.4501</td>
<td>0.5751</td>
<td>0.7001</td>
<td>0.8251</td>
<td>0.95</td>
</tr>
<tr>
<td>a=0.05</td>
<td>0.4522</td>
<td>0.5764</td>
<td>0.7009</td>
<td>0.8256</td>
<td>0.9505</td>
</tr>
<tr>
<td>a=0.07</td>
<td>0.455</td>
<td>0.5778</td>
<td>0.7018</td>
<td>0.8262</td>
<td>0.9509</td>
</tr>
</tbody>
</table>

V. COSTS OF IGNORING CUSTOMER STRATEGIC BEHAVIOR

An interesting question is: what happens if the firm ignores customer strategic behavior and makes decisions based on the belief that customers think \( q=1 \). In this section, we will answer this question.

By Part (a) of Proposition 4.1, we have

\[ p^*(1) > p^*(q), \text{ for any } 0<q<1, \]

which shows that the online retailer will price higher if it ignores customer strategic behavior. Let \( \bar{\pi}(q) := \pi(p^*(1), q) \) be the online retailer’s single-period profit with TSP \( q \) when ignoring customer strategic behavior. Similarly as in Section IV, let \( \bar{V}_{i+1}(q_i) = 0 \) for any \( q \in [q, \bar{q}_n] \) and

\[ \bar{V}_i(q_i) = \max_{q \in [q, \bar{q}_i]} \left\{ -K_i \delta(q - q_i) + \bar{\pi}(q) + \beta \bar{V}_{i+1}(q_i) \right\}, \quad i = 1, 2, \ldots, n. \]

Then \( \bar{V}_i(q_i) \) stands for the online retailer’s total discounted profit from Period \( i \) to Period \( n \) with TSP \( q_i \) when ignoring customer strategic behavior. Denote the online retailer’s profit loss rate of ignoring customer strategic behavior by
\[ r = \frac{V_i(q) - \hat{V}_i(q)}{V_i(q)}. \]

Next we conduct some numerical examples to investigate how large \( r \) is and how \( r \) changes with model parameters. Let \( F(x) \) be the CDF of uniform distribution over \([0, A]\), \( u(x) = 1 - \exp(ax) \), \( n = 3 \), \( N = 1000 \), \( K_i = 1000 \), \( i = 1, 2, 3 \), \( q_0 = 0.5 \), \( \bar{q}_1 = 0.6 \), \( \bar{q}_2 = 0.7 \), \( \bar{q}_3 = 0.8 \), \( \beta = 0.9 \). The results are demonstrated in Figure 1.

**FIGURE I**

\( r \) V.S. MODEL PARAMETERS

From Figure 1, we have the following observations: (1) the profit loss rate of ignoring customer strategic behavior can be significantly high (\( r \) can be up to 65% in some cases); (2) the profit loss rate of ignoring customer strategic behavior is more significant if customer valuations for the product (\( A \)) are smaller; customer transaction cost (\( h \)) is lower; or if customers are more risk averse (i.e., \( a \) is larger). This indicates that the negative effect of ignoring customer strategic behavior is substantial. To alleviate the negative effect of ignoring customer strategic behavior, the firm should (1) increase customer valuations for the product by, for example, better product design or
more impressive advertisement; (2) lower customer transaction cost by, for example, providing better navigation aids. Besides, when facing less risk averse customers, the negative effect of ignoring customer strategic behavior is smaller.

VI. EXTENSIONS AND VARIATIONS

In this section, we provide some extensions and variations. Analyses of these extensions and variations show the robustness of the main results above.

A. Concave Upgrading Cost

In previous sections, we assume the web system upgrade in a period only incurs a fixed cost. Here we consider a more general cost structure. Let \( C_i(q - q_i) \) be the cost of increasing TSP from \( q_i \) to \( q \) in Period \( i \) \((i = 1, 2, \cdots, n)\). We assume \( C_i(\cdot) \) is an increasing and concave function. This assumption represents the benefits of economic scale: the marginal cost of upgrading becomes lower when the increased magnitude of TSP is larger. Some common cost functions fall into this category, such as \( K_i\delta(q - q_i) + b_i(q - q_i) \), where \( K_i > 0, b_i > 0 \). For tractability, we only consider risk neutral customers and uniform distribution for customer valuations, i.e., \( u(x) = x \) and \( F(x) \) is the CDF of uniform distribution over \([0, A]\). To avoid trivial cases, we assume \( A > h/q \). In this case, \( x(q, h) = h/q \) and

\[
\pi^*(q) = \frac{N \left( A^2q - 2Ah + \frac{h^2}{q} \right)}{4A}.
\]  

(12)

It can be easily verified that \( \pi^*(q) \) in Eqn. (12) is increasing and convex with respect to \( q \) when \( A > h/q \).

Let \( V_i(q_i) \) denote the optimal total profit from Period \( i \) to Period \( n \). Then the firm faces the following dynamic program:

\[
V_i(q_i) = \max_{q \in [q_i, q]_i} \left\{ -C_i(q - q_i) + \pi^*(q) + \beta V_{i+1}(q) \right\}, \quad i = 1, 2, \cdots, n.
\]
The following proposition is an analogy of Proposition 4.3 and reduces the decision space of this problem significantly.

**Proposition 6.1:** Suppose \( u(x) = x \) and \( F(x) \) is the CDF of uniform distribution over \([0, A]\). For \( i = 1, 2, \ldots, n \),

(a) \( V_i(q_i) \) defined in (13) is convex with respect to \( q_i \).

(b) The optimal TSP \( q_i^* \in \{q_i, \bar{q}_i\} \).

By similar arguments, the main findings in Sections IV and V remain true for the general concave upgrading cost.

**B. Other Extensions and Variations**

In section IV, we consider the pricing decision of the retailer when all the customers are strategic. In fact, in real world there do exist several nonstrategic customers. We give an extended modal to characterize the pricing decision when both strategic and nonstrategic customers exist.

Using \( \lambda (0 \leq \lambda \leq 1) \) as the proportion of strategic customers, then \( 1 - \lambda \) is the proportion of nonstrategic customers. The demand for the strategic customers is \((1 - \lambda)NF(p)\) and \((1 - \lambda)NF(p)\) for the nonstrategic customers(nonstrategic customers will ignore the invisible cost). Then the firm’s expected profit in a single period with TSP \( q \), selling price \( p \) and proportion \( \lambda \) is

\[
\pi(p, q, \lambda) = \lambda pqNF(p + x(q, h)) + (1 - \lambda) pqNF(p).
\]  

(13)

Apparently, given \( p \) and \( q \), \( \pi(p, q, \lambda) \) is a decreasing function of \( \lambda \). That means the retailer will get higher profit when more customers are nonstrategic. Taking derivative of \( \pi(p, q, \lambda) \) with respect to \( p \), we have

\[
\frac{\partial \pi(p, q, \lambda)}{\partial p} = \lambda qNF(p + x(q, h))(1 - pr(p + x(q, h))) + (1 - \lambda)qNF(p)(1 - pr(p)).
\]  

(14)
According to Lemma 4.1, there exists only one $p^*$ such that $1 - p^* r(p^* + x(q, h)) = 0$. Similarly, there exist only one $p^{**}$, such that $1 - p^{**} r(p^{**}) = 0$. Since $r$ is an increasing function, for $p < p^*$, the function $\frac{\partial \pi(p, q, \lambda)}{\partial p}$ is decreasing and bigger than 0. On the other hand, for $p > p^{**}$, the function $\frac{\partial \pi(p, q, \lambda)}{\partial p}$ is increasing and less than 0. Therefore the optimal price $p^*$ is in the interval $[p^*, p^{**}]$.

When $F(x)$ is the CDF of uniform distribution over $[0, A]$, $p'(q, h, \lambda) = \frac{A - \lambda x(q, h)}{2}$ is the only point satisfying $\frac{\partial \pi(p, q, \lambda)}{\partial p} = 0$. From Proposition 3.2, 3.3, $p'(q, h, \lambda)$ is increasing with respect to $q$, decreasing with respect to $h$, $\lambda$ and customers’ degree of risk aversion. This result is consistent with the main result in section IV.

In previous sections, we assume that customers are able to correctly expect TSP. Some may argue this assumption may not hold in reality, so we drop this assumption and consider an adaptive learning model here. Specifically, in Period $i$ ($i = 1, 2, \cdots, n$), customers have a homogeneous belief over TSP $\hat{q}_i$, and this belief is updated based on the belief and the actual TSP of the previous period:

$$\hat{q}_i = \theta q^*_i + (1 - \theta)\hat{q}_{i-1},$$

where $q^*_i$ and $\hat{q}_{i-1}$ are the actual TSP and the belief in Period $i - 1$ respectively, and $\theta \in [0,1]$ is customer learning speed. The timing of events is the same as in Section IV. By similar arguments, main findings and managerial insights in Sections IV and V remain true for this adaptive learning model.

Besides, main findings are also true for random upgrading costs, whose
distributions are assumed to be known by the online retailer.

VII. CONCLUDING REMARKS AND FUTURE RESEARCH

In this paper, we consider pricing and web system upgrading problems for an online retailer who faces a group of strategic customers. Due to various website issues, there is a possibility of transaction failure when a customer purchases the product through Internet. The strategic customers can anticipate the probability of transaction failure and decide whether to purchase the product based on their beliefs on transaction success probability. We obtain the following results in this paper:

(1) We characterize the optimal purchasing policy for customers: there exists a threshold such that a customer will buy the product if his valuation for the product is above the threshold, and will not buy otherwise. Furthermore, the threshold is lower if the transaction success probability is higher, the customer transaction cost is lower, or if customers are less risk averse.

(2) We derive the optimal price of each period and identify the optimal policy for web system upgrading. The optimal policy for the online retailer to upgrade its web system is a threshold policy: There exists a threshold for each period such that the online retailer should upgrade its web system to the state of art (i.e., achieve the highest available TSP) if the current TSP is below the threshold, and should not upgrade otherwise. The threshold (total discounted profit) increases as customer transaction cost decreases, customer valuations for the product become higher, or customers become more (less) risk averse.

(3) The online retailer tends to price higher if they ignore customer strategic behavior. The cost of ignoring customer strategic behavior can be significantly high (sometimes the profit loss rate of ignoring customer strategic behavior can be up to 65%). To alleviate the negative effect of ignoring customer strategic behavior, the online retailer should (1) increase customer valuations for the product by better
product design or more impressive advertisement; or (2) lower customer transaction cost by providing better navigation aids. Besides, when facing less risk averse customers, the negative effect of ignoring customer strategic behavior is lower.

These results are robust since they remain true for several extensions and variations of the model.

There are several interesting directions for future research. First, in this paper we assume customers’ valuations follow a distribution with increasing hazard rate, and for the situation that the upgrade cost \( C(q - q_i) \) is a general concave function, our discussions are limited to the risk-neutral customers (i.e., \( u(x) = x \)) and the uniform valuation distribution. It will be worthwhile to relax these limitations to get the optimal upgrading policies. Second, this paper focuses on pricing and web system upgrading problems of a monopolist. It is more interesting to study similar problems when online retailers face competition from rivals. Second, this paper assumes that the transaction success probability does not depend on the number of customers who purchase the product. It is more interesting to study similar problems when the transaction success probability depends on the number of customers who try to purchase the product (e.g., the transaction success probability decreases when the number of customers who try to purchase the product increases).

**APPENDIX**

**Proof of Proposition 3.1.** Let \( x = v - p \). Since \( u(\cdot) \) is an increasing function, then

\[
f(x) = qu(x - h) + (1 - q)u(-h)
\]

is also increasing with respect to \( x \). In addition, since \( u(0) = 0 \) and \( u(+\infty) = +\infty \), we have \( f(h) = (1-q)u(-h) < u(0) \) and \( \lim_{x \to +\infty} f(x) > u(0) \). Thus, \( x(q,h) \) is the unique solution of

\[
qu(x - h) + (1 - q)u(-h) = u(0) = 0
\]
according to the analysis just above Inequality (1), and \( x(q,h) > h \). In Inequality (1), “≥” holds if \( v \geq p + x(q,h) \), and “<” holds if \( v < p + x(q,h) \). Therefore, the purchasing decisions of customers described in Proposition 3.1 follows.

**Proof of Proposition 3.2.** Recall the definition of \( x(q,h) \) in Eqn. (2), for any \( 0 < q_1 < q_2 \leq 1 \), we have

\[
\frac{u(x(q_1,h) - h)}{u(-h)} = 1 - \frac{1}{q_1} < 1 - \frac{1}{q_2} = \frac{u(x(q_2,h) - h)}{u(-h)},
\]

which implies \( u(x(q_1,h) - h) > u(x(q_2,h) - h) \). Thus, \( x(q_1,h) > x(q_2,h) \) since \( u(\cdot) \) is an increasing function. Therefore, \( x(q,h) \) is decreasing with respect to \( q \).

Similarly, it can be proved that \( x(q,h) \) is increasing with respect to \( h \).

**Proof of Proposition 3.3.** For any \( x \geq h \). Consider function \( g(x) = u(x)/w(x) \). Taking first derivative of \( g(x) \), we have

\[
g'(x) = \frac{w(x)u'(x) - u(x)w'(x)}{(w(x))^2} = \frac{u'(x)\varphi(u(x)) - u(x)\varphi'(u(x))}{(w(x))^2}.
\]

Since \( \varphi(\cdot) \) is an increasing concave function with \( \varphi(0) = 0 \), then

\[
\varphi(u(x)) + [0 - u(x)]\varphi'(u(x)) \geq \varphi(0) = 0,
\]

which leads to \( g'(x) \geq 0 \), indicating that \( g(x) \) is an increasing function. Thus,

\[
\frac{u(x-h)}{w(x-h)} \geq \frac{u(-h)}{w(-h)},
\]

which implies

\[
\frac{u(x-h)}{u(-h)} \leq \frac{w(x-h)}{w(-h)}.
\]

Since

\[
\frac{w\left(x''(q,h) - h\right)}{w(-h)} = 1 - \frac{1}{q} = \frac{u\left(x''(q,h) - h\right)}{u(-h)} \leq \frac{w\left(x''(q,h) - h\right)}{w(-h)},
\]

then \( x''(q,h) \geq x''(q,h) \). This ends the proof.
Proof of Lemma 4.1. Part (a). Taking derivative of $\pi(p,q)$ with respect to $p$, we have

$$\frac{\partial \pi(p,q)}{\partial p} = qNF(p + x(q,h)) \left[ 1 - p \frac{f(p + x(q,h))}{F(p + x(q,h))} \right].$$

Since $qNF(p + x(q,h)) > 0$ (else there is no demand, which is a trivial case) and $F(x)$ has increasing failure rate, then $\frac{\partial \pi(p,q)}{\partial p} = 0$ has a unique solution, which satisfies (5). Denote this solution by $p^*(q)$. Since

$$\frac{\partial^2 \pi(p,q)}{\partial p^2} = -qNF(p^*(q) + x(q,h)) \frac{\partial \left[ pr(p + x(q,h)) \right]}{\partial p} < 0,$$

then $p^*(q)$ is a local maximum. Note that $\frac{\partial \pi(p,q)}{\partial p} = 0$ only has a unique solution $p^*(q)$. Then $\pi(p,q)$ is unimodal with respect to $p$ and $p^*(q)$ is the global maximum over $[0, +\infty)$. This ends the proof. ■

Proof of Proposition 4.1. Part (a). We only provide the proof of the statement that $p^*(q)$ is increasing with respect to $q$, since other proofs are similar. For any $0 < q_1 < q_2 < 1$, we have $x(q_1, h) > x(q_2, h)$ by Proposition 3.2. Thus, since $F(x)$ has increasing failure rate, then

$$p^*(q_1) \frac{f(p^*(q_1) + x(q_2, h))}{F(p^*(q_1) + x(q_2, h))} \leq p^*(q_2) \frac{f(p^*(q_2) + x(q_1, h))}{F(p^*(q_2) + x(q_1, h))}$$

$$= p^*(q_2) \frac{f(p^*(q_2) + x(q_2, h))}{F(p^*(q_2) + x(q_2, h))},$$

which implies that $p^*(q_1) \leq p^*(q_2)$.

Part (b). We only provide the proof of the statement that $\pi^*(q)$ is increasing with respect to $q$, since other proofs are similar. For any $0 < q_1 < q_2 < 1$, by Proposition 3.2 and the definition of $\pi(p,q)$ in Eqn. (0*), we have $\pi(p,q_1) < \pi(p,q_2)$. Thus,
\[ \pi^*(q_i) = \pi(p^*(q_i), q_i) < \pi(p^*(q_i), q_1) \leq \pi(p^*(q_2), q_2) = \pi^*(q_2), \]
which implies that \( \pi^*(q) \) is increasing with respect to \( q \). ■

**Proof of Proposition 4.2.** By \( F_x(\cdot) \leq_{hr} F_y(\cdot) \) and the definitions of \( p_{F_x}^* \) and \( p_{F_y}^* \), we have
\[
p_{F_x}^* \cdot r_F(p_{F_x}^* + x(q,h)) \leq p_{F_x}^* \cdot r_F(p_{F_x}^* + x(q,h)) = p_{F_y}^* \cdot r_F(p_{F_y}^* + x(q,h)).
\]
Thus, \( p_{F_y}^* \geq p_{F_x}^* \) since \( F_Y(v) \) has increasing hazard rate. ■

**Proof of Proposition 4.3.** We prove this proposition by contradiction. Suppose that there exists some \( k \in \{1, 2, \ldots, n\} \) such that \( q_{k-1}^* < q_k^* < \bar{q}_k \). Let
\[
i_0 = \begin{cases} n+1, & \text{if } q_i^* = q_k^* \text{ for all } i > k, \\ \min \{i > k : q_i^* > q_k^*\}, & \text{otherwise.} \end{cases}
\]
We construct \( \tilde{Q} = (\tilde{q}_1, \tilde{q}_2, \ldots, \tilde{q}_n) \) as follows: \( \tilde{q}_i = q_i^* \) for all \( i \in [1, k-1] \cup [i_0, n] \); \( \tilde{q}_i = q_i^* + \Delta q \) for all \( i \in [k, i_0 - 1] \), where \( \Delta q > 0 \) is a sufficiently small constant such that \( \tilde{q}_i \leq \bar{q}_i \). We only need to prove the total discounted profit corresponding to \( \tilde{Q} \) is greater than that corresponding to \( Q^* \). First, it is easy to check that \( \tilde{Q} \) incurs the same cost as \( Q^* \). Second, since \( \pi^*(q) \) is increasing with respect to \( q \), then \( \pi^*(\tilde{q}_i) > \pi^*(q_i^*) \). Thus, in Periods \( i \in [k, i_0 - 1] \), \( \tilde{Q} \) generates higher profit than \( Q^* \). This ends the proof. ■

**Proof of Proposition 4.4.** We prove this proposition by induction. First, let’s consider Period \( n \). By definitions of \( J_i^*(q_i) \) and \( J_i^{**}(q_i) \) in Eqns. (8) and (9),
\[
J_n^u(q_n) = -K_n + \pi^*(\bar{q}_n), \quad J_n^{**}(q_n) = \pi^*(q_n).
\]
Apparently, \( J_n^u(q_n) \) is regardless of \( q_n \) and \( J_n^{**}(q_n) \) is increasing in \( q_n \). Thus, \( J_n(q_n) = J_n^u(q_n) - J_n^{**}(q_n) \) is decreasing in \( q_n \). Besides, since \( V_n(q_n) = \max \{J_n^u(q_n), J_n^{**}(q_n)\} \), then \( V_n(q_n) \) is also increasing in
If \( \max_{q \in [q_n, \bar{q}_n]} J_n(q) < 0 \), it is not profitable to upgrade the web system for any \( q \in [q_n, \bar{q}_n] \). In this case, it is optimal not to upgrade the web system. Besides, by definition (12), \( T = 0 < q_n \), which indicates Part (e) is true. If \( \max_{q \in [q_n, \bar{q}_n]} J_n(q) \geq 0 \), we differ the proof in two cases: (1) \( J_n(q) \geq 0 \) for any \( q \in [q_n, \bar{q}_n] \); (2) \( J_n(q) < 0 \) for some \( q \in [q_n, \bar{q}_n] \). For Case (1), it is always profitable to upgrade the web system to achieve TSP \( \bar{q}_n \) (except for the case \( q_n = \bar{q}_n \)). By the definition in Eqn. (12), \( T = \bar{q}_n \). Note that \( q_n \leq \bar{q}_{n-1} \leq \bar{q}_n = T \) always holds, then Part (e) follows. For Case (2), by the definition in Eqn. (12), \( T \) satisfies \( J_n(T) = 0 \). Since it is optimal to upgrade the web system if \( J_n(q_n) > 0 \) and not to upgrade otherwise. Note that \( J_n(q_n) \) is decreasing in \( q_n \), then Part (e) is true.

Suppose this proposition holds for \( i+1 \). Now we consider \( i \). According to Eqn. (8), we have \( J_i^u(q_i) \) is regardless of \( q_i \). By Eqn. (9) and the induction assumption that \( V_{i+1}(q) \) is increasing in \( q \), \( J_i^{mu}(q_i) \) is increasing in \( q_i \). Thus, obviously \( J_i(q_i) = J_i^u(q_i) - J_i^{mu}(q_i) \) is decreasing in \( q_i \) and \( V_i(q_i) = \max \{ J_i^u(q_i), J_i^{mu}(q_i) \} \) is increasing in \( q_i \). The above arguments prove Parts (a)-(d). The proof for Part (e) is similar as Period \( n \). ■

**Proof of Proposition 6.1.** We prove this proposition by induction. For Period \( n \),

\[
V_n(q_n) = \max_{q \in [q_n, a_n]} \left\{ -C_n(q - q_n) + \pi^*(q) \right\}.
\]

Since \( \pi^*(q) \) is convex and \( C_n(\cdot) \) is concave, then \( -C_n(q - q_n) + \pi^*(q) \) is convex with respect to \( q \), thus achieve its maximum on the boundary: \( q_n^* \in \{q_n, \bar{q}_n\} \). Therefore,
\[ V_n(q_n) = \max_{q \in (q_n, \bar{q}_n)} \left\{ -C_n(q - q_n) + \pi^*(q) \right\} \]

\[ = \max \left\{ -C_n(0) + \pi^*(q_n), -C_n(\bar{q}_n - q_n) + \pi^*(\bar{q}_n) \right\}. \]

Note that both \(-C_n(0) + \pi^*(q_n)\) and \(-C_n(\bar{q}_n - q_n) + \pi^*(\bar{q}_n)\) are convex with respect to \(q_n\). Then \(V_n(q_n)\) is a convex function.

Assume the proposition is true for Period \(i+1\), we need to prove it holds for Period \(i\). By Eqn. (13), \(-C_i(q - q_i) + \pi^*(q) + \beta V_{i+1}(q)\) is convex with respect to \(q\) since \(-C_i(q - q_i), \pi^*(q)\) and \(V_{i+1}(q)\) are all convex with respect to \(q\). Thus, the optimal point is on the boundary: \(q^*_i \in \{q_i, \bar{q}_i\}\). Furthermore,

\[ V_i(q_i) = \max_{q \in [q_i, \bar{q}_i]} \left\{ -C_i(q - q_i) + \pi^*(q) + \beta V_{i+1}(q) \right\} \]

\[ = \max \left\{ -C_i(0) + \pi^*(q_i) + \beta V_{i+1}(q_i), -C_i(q_i - q_i) + \pi^*(q_i) + \beta V_{i+1}(q_i) \right\}. \]

Since both \(-C_i(0) + \pi^*(q_i)\) and \(-C_i(q_i - q_i) + \pi^*(q_i) + \beta V_{i+1}(q_i)\) are convex with respect to \(q_i\), then \(V_i(q_i)\) is convex. This completes the proof. ■

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