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Price discount based on early order commitment in a single manufacturer–multiple retailer supply chain

Jinxing Xie [a,*], Deming Zhou [b,1], Jerry C. Wei [c,2], Xiande Zhao [b,3]

[a] Department of Mathematical Sciences, Tsinghua University, Beijing 100084, China
[b] Department of Decision Sciences and Managerial Economics, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong
[c] Mendoza College of Business, The University of Notre Dame, Notre Dame, IN 46556, USA

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ABSTRACT

Early order commitment (EOC) is a strategy for supply chain coordination, wherein the retailer commits to purchasing from a manufacturer a fixed order quantity a few periods in advance of the regular delivery lead time. In this paper, we formulate and analyze the EOC strategy for a decentralized, two-level supply chain consisting of a single manufacturer and multiple retailers, who face external demands that follow an autocorrelated AR(1) process over time. We characterize the special structure of the optimal solutions for the retailers' EOC periods to minimize the total supply chain cost and discuss the impact of demand parameters and cost parameters. We then develop and compare three solution approaches to solving the optimal solution. Using this optimal cost as the benchmark, we investigate the effectiveness of using the wholesale price-discount scheme for the manufacturer to coordinate this decentralized system. We give numerical examples to show the benefits of EOC to the whole supply chain, examine the efficiency of the discount scheme in general situation, and provide the special conditions when the full coordination is achieved.

1. Introduction

It is believed that coordination between members of the supply chain is vital for effective supply chain management. By coordinating activities across company boundaries, significant benefits can be achieved for the partners in a supply chain. In previous research on supply chain coordination, it is suggested to share real-time demand data collected at the points of sale with upstream suppliers (Lee et al., 2000; Cachon and Fisher, 2000; Raghunathan, 2001), or to use a collaborative, centralized forecasting mechanism (Xu et al., 2001; Aviv, 2001), or to adopt a vendor managed inventory in which the vendor is authorized to manage the replenishment process (Fry et al., 2001; Aviv, 2002). Noticing that it is to the seller's advantage to encourage buyers to commit to their purchase orders before actual production begins, another stream of research focusing on order commitments also attracts great interest from both academic researchers and practitioners. Fisher and Raman (1996) help a fashion skiwear firm use early orders to revise and improve sales forecasts. Iyer and Bergen (1997) report that explicit total quantity volume, or dollar volume commitments, from retailers helped a supplier procure materials, schedule production, and better utilize capacity.

In this paper, early order commitment (EOC) means that a retailer commits to purchase from a manufacturer a fixed order quantity a few periods in advance of the regular delivery lead time. For instance, if a retailer uses EOC policy with t time periods in advance of its regular delivery lead time L_i, she will place the order with $L_i + t$ time periods earlier, while the manufacturer still only needs to fulfill the order L_i periods earlier. Zhao et al. (2002) conduct extensive simulation studies on the effect of EOC on supply chain performance under various operational conditions, including demand pattern, forecast errors, cost structure, number of retailers, and capacity cushion. They found that EOC produces substantial cost savings for the manufacturer but increases costs for the retailer. To maximize the cost savings from EOC, the members at different levels of the supply chain should make the tradeoff based on a careful evaluation of both the costs and the benefits. Zhao et al. (2007) propose such an analytical model to quantify the effects of EOC on the performance of a simple two-level supply chain consisting of a manufacturer and a retailer. The end demand is a nonstationary autoregressive AR(1) process which is prevalent in the high-technology and grocery industries (Lee et al., 2000). In consumer products, sales incentives can cause high correlations between successive monthly demands and between demands for

* Corresponding author. Tel.: +86 10 6278 7812; fax: +86 10 6278 5847.
E-mail addresses: jxie@math.tsinghua.edu.cn, xiejinxing@tsinghua.org.cn (J. Xie), deming@baf.msmail.cuhk.edu.hk (D. Zhou), jwei@nd.edu (J.C. Wei), xiande@baf.msmail.cuhk.edu.hk (X. Zhao).
[1] Tel.: +852 2609 7768; fax: +852 2603 6840.
[2] Tel.: +1 574 631 5460; fax: +1 574 631 5255.
[3] Tel.: +852 2609 7650; fax: +852 2603 6840.

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an item at different locations (Erkip et al., 1990). Studying an AR(1) demand process increases the complexity of the model but makes the model more realistic than using the typical assumption of independent identical demand found in the literature. In Zhao et al. (2007) a decision rule was developed to determine whether EOC can benefit the supply chain and accordingly choose the optimal period for early commitment.

To entice buyers to commit orders earlier, two types of incentive are common. In some industries, suppliers provide price discounts or better payment terms to buyers in exchange for the commitment of their purchase orders. For example, Gilbert and Ballou (1999) conduct an analysis of a steel distribution supply chain and quantify the maximum discount that can be offered to consumers who commit to orders in advance. Cvsa and Gilbert (2002) examine the tradeoff between early order commitment and order postponement in the context of competition. Using a game theory model involving a monopolistic supplier without capacity constraint and two duopolistic buyers, they find that a supplier can influence the form of competition in the downstream market. Zhao et al. (2007) develop a rebate scheme for a decentralized, single manufacturer–single retailer channel to induce the retailer's participation in EOC and help determine how the two parties should split the net savings from practicing EOC. In this paper, we extend the model of Zhao et al. (2007) to a single manufacturer–multiple retailer case, which is more realistic because a supplier usually uses more than one outlet to distribute its products to customers. At the same time, the multiple retailer environment makes the supply chain coordination model much more complicated and challenging. First, we formulate the cost model for the coordinated supply chain and develop the properties of its optimal solution. We then examine three different searching approaches to derive the optimal or near optimal solution for the supply chain and provide a special case when one of the heuristics can always lead to the optimal solution. Furthermore, we develop a wholesale price-discount scheme that allows the supply chain partners to share the net savings resulting from EOC and thus can induce the retailers to voluntarily participate. We also analyze the efficiency of this price-discount scheme and provide the numerical examples and special cases when the decentralized supply chain coordination can be achieved under the discount scheme.

The paper is organized as follows. The basic model of the ideal coordinated supply chain is formulated in Section 2, and three approaches to solving the optimal solution are discussed. Section 3 presents the game-theoretic analysis when the manufacturer provides EOC-based price discount to induce the retailers to practice EOC. Final remarks are concluded in Section 4.

2. The basic model

2.1. Basic assumptions

The basic assumptions of the model in this paper are similar to those adopted by Lee et al. (2000) and Zhao et al. (2007) for a single manufacturer–retailer dyad. However, we expand the supply chain to include a single manufacturer who sells to N geographically dispersed, non-competing (thus independent) retailers. All the parties in this supply chain are separately owned. Each retailer faces an external demand for a single product, and the demand is assumed to be a simple, autocorrelated, AR(1) process, i.e., the demand in period t for retailer i is

$$D_{it} = d_i + \rho_i D_{i,t-1} + \varepsilon_{i,t}, \quad i = 1, 2, \ldots, N, \tag{1}$$

where process mean $d_i > 0$, and correlation coefficient $0 < \rho_i < 1$. For a given retailer i, the random element of demand at period t, $\varepsilon_{i,t}$, follows a normal distribution with mean zero and variance σ_i^2, which is assumed to be i.i.d. over time. Furthermore, for different retailers i and j, $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ are assumed to be independent random variables. We also assume that σ_i is significantly smaller than d_i, so that the probability of a negative demand is negligible. Lee et al. (2000) show that the assumption of an AR(1) demand process with known characteristic parameters is reasonable in real world supply chain management when demand information is passed from the retailer to the manufacturer. Raghunathan (2001) points out that even when demand information is not shared from the retailers to the manufacturer, the manufacturer can also estimate the demand process and the related parameters using some forecasting approaches. Therefore, in this research we assume that demand information is shared between the retailers and the manufacturer, and the demand process and its characteristic parameters are assumed to be common knowledge among the manufacturer and the retailers: that is, all the retailers and the manufacturer know the demand distribution in Eq. (1) and the specific values of the parameters d_i, ρ_i, and σ_i for all retailer i. Also on a one-supplier, multiple retailer setting, Cachon (2001) studies the competition between the supplier and each retailer in a supply chain game model. The game model is based on reorder points and does not consider AR(1) demand and EOC.

We consider a periodical review system with a make-to-stock policy in which each site reviews its inventory position and replenishes its inventory from the upstream site at the end of every period. The manufacturing lead time for the manufacturer (including the lead time for the replenishment of raw materials from exogenous suppliers) is a constant L_0, and the delivery lead time from the manufacturer to retailer i is a constant L_i ($i = 1,2,\ldots,N$). Each retailer may choose to use or not to use EOC policy when making ordering decisions. The EOC period is the extra period that a retailer can choose to place the order earlier than the regular delivery lead time. If the manufacturer does not have enough stock to fill the orders, we assume that the manufacturer will meet the shortfall by obtaining additional units from an external source. Thus, the manufacturer guarantees supply to the retailers but bears the expedite cost. Similar assumptions had been used in previous studies of decentralized systems, including Bourland et al. (1996), Lee and Whang (1999), Gavirneni et al. (1999), Lee et al. (2000), and Raghunathan (2003). This assumption eliminates the concern of shortage delay and enables the practice of EOC. We also assume that no fixed ordering cost is incurred when placing the order, and that unit inventory holding cost and backorder cost are stationary over time. Both the retailers and the manufacturer use an order-up-to policy, a periodic reviewing policy which is optimal for a stochastic inventory system without a fixed ordering cost, to make their ordering (or manufacturing) decisions with the review interval being one period (e.g., a daily review).

Notice that although the basic setting of this model is similar to the setting of the traditional multi-echelon inventory systems with retailers, there exist several significant differences that distinguish our model from the multi-echelon inventory systems discussed by Zipkin (2000), Langenhoff and Zijm (1990), etc. First of all, our focus is on the relationship between the manufacturer and the multiple retailers, which is essentially a decentralized system, while multi-echelon inventory systems usually discuss centralized decision making which is equivalent to a vertically integrated supply chain. Second, we explicitly discuss the shortage cost incurred at the manufacturer, while in the multi-echelon inventory literature, backlog or shortage cost between the echelon players is typically neglected since it is an internal cost (Lee and Whang, 1999). Third, we allow the differences in all the parameters among the downstream retailers, and thus discuss the ordering decisions of the players in a general distribution system. Finally, we use the optimal cost for the coordinated supply chain as the benchmark to discuss the interactions between early ordering commitment and incentives scheme in the general decentralized supply chain system. Our focus is to provide an effective and efficient scheme to induce possible supply chain coordination between the players who seek to minimize their own costs.

2.2. Retailers' holding and shortage costs

Consider the ordering decision for retailer i ($i = 1, 2, \ldots, N$) at any period t. Denote the unit holding cost and backorder cost per period for retailer i as h_i and p_i, respectively. Besides, let $\Phi(\bullet)$ be the cumulative distribution function of the standard normal distribution and define

$$k_i = \Phi^{-1}\left(\frac{p_i}{h_i + p_i}\right). \tag{2}$$

When retailer i uses the order-up-to policy and place her order x_i periods ahead, the uncertainty of the demand during periods $[t+1, t+L_i+x_i+1]$ should be considered in period t's order. According to the demand process of Eq. (1), the demand during periods $[t+1, t+L_i+x_i+1]$ is normally distributed with the variance of

$$v_i(x_i) = \frac{\sigma_i^2}{(1-\rho_i)^2}\sum_{j=1}^{L_i+x_i+1}(1-\rho_i^j)^2, \tag{3}$$

which is independent of t. According to Lee et al. (2000) and Zhao et al. (2007), retailer i's expected inventory holding and shortage costs per period can be expressed as

$$c_i(x_i) = r_i\sqrt{v_i(x_i)} = r_i\frac{\sigma_i}{1-\rho_i}\sqrt{\sum_{j=1}^{L_i+x_i+1}(1-\rho_i^j)^2}, \tag{4}$$

where $r_i = (h_i + p_i)\eta(k_i) + h_i k_i$ represents the cost structure of retailer i, in which $\eta(\bullet)$ is the right loss function for the standard normal distribution, i.e.,

$$\eta(x) = \int_x^\infty (z-x)d\Phi(z). \tag{5}$$

2.3. Manufacturer's holding and shortage cost

Similarly, we can consider the ordering decisions for the manufacturer. Denote the unit holding cost and backorder cost period for the manufacturer as h_0 and p_0, respectively, and define

$$k_0 = \Phi^{-1}\left(\frac{p_0}{h_0 + p_0}\right). \tag{6}$$

Throughout this paper, we assume that the EOC period x_i is less or equal to the manufacturing lead time L_0 plus one extra period, i.e., $0 \leqslant x_i \leqslant L_0 + 1$. This assumption is justified because committing to orders more than $L_0 + 1$ periods ahead increases retailer i's cost without providing extra benefit to the manufacturer. Note that $x_i = 0$ corresponds to the case where retailer i does not commit orders to the manufacturer in advance. In contrast, $x_i = L_0 + 1$ corresponds to the case where demand uncertainty from retailer i is completely eliminated for the manufacturer because the manufacturer can react to demand changes beyond his manufacturing lead time L_0.

For the order-up-to policy with lead time, we need to consider the uncertainty of the demand during the whole $L_0 + 1$ periods, i.e., all the retailers' orders which should be shipped in the following $L_0 + 1$ periods. Notice that due to x_i periods of EOC, the cumulative quantity of the orders which should be shipped to retailer i during these $L_0 + 1$ periods only has the uncertainty of $L_0 - x_i + 1$ periods. According to Zhao et al. (2007), this cumulated quantity of $L_0 + 1$ periods is normally distributed with the variance of $\frac{\sigma_i^2}{(1-\rho_i)^2}\sum_{j=L_i+x_i+2}^{L_i+L_0+2}(1-\rho_i^j)^2$. Since the manufacturer's total demand consists of the orders from all the N independent retailers, his total demand during periods $[t+1, t+L_0+1]$ is normally distributed with the variance of

$$v_0(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N}\frac{\sigma_i^2}{(1-\rho_i)^2}\sum_{j=L_i+x_i+2}^{L_i+L_0+2}(1-\rho_i^j)^2. \tag{7}$$

Denote $x = (x_1, x_2, \ldots, x_N)$. According to Lee et al. (2000) and Zhao et al. (2007), the manufacturer's expected inventory holding and shortage costs per period can be expressed as

$$\begin{aligned} c_0(x) &= r_0\sqrt{v_0(x_1, x_2, \ldots, x_N)} \\ &= r_0\sqrt{\sum_{i=1}^{N}\left(\frac{\sigma_i}{1-\rho_i}\right)^2\sum_{j=L_i+x_i+2}^{L_i+L_0+2}(1-\rho_i^j)^2}, \end{aligned} \tag{8}$$

where $r_0 = (h_0 + p_0)\eta(k_0) + h_0 k_0$ represents the cost structure of the manufacturer.

2.4. Supply chain's holding and shortage cost

Summarizing Eqs. (4) and (8), the expected holding and shortage cost per period for the whole supply chain is

$$\begin{aligned} SC(x) &= c_0(x) + \sum_{i=1}^{N}c_i(x_i) \\ &= r_0\sqrt{\sum_{i=1}^{N}\left(\frac{\sigma_i}{1-\rho_i}\right)^2\sum_{j=L_i+x_i+2}^{L_i+L_0+2}(1-\rho_i^j)^2} + \sum_{i=1}^{N}r_i\frac{\sigma_i}{1-\rho_i} \\ &\quad \times\sqrt{\sum_{j=1}^{L_i+x_i+1}(1-\rho_i^j)^2}. \end{aligned} \tag{9}$$

When there exists only a single retailer ($N = 1$), the optimal EOC periods should be either 0 (without EOC policy) or $L_0 + 1$ (order as earlier as possible) in order to minimize the supply chain's cost (Zhao et al., 2007). The following theorem reveals that this property still holds for the supply chain with multiple independent retailers. Hereon, all the proofs of Theorem and Propositions are provided in the Supplementary material (Appendix A).

Theorem 1. *In order to minimize the supply chain's cost expressed as Eq. (9), the optimal EOC period for each retailer should be either 0 or $L_0 + 1$.*

Suppose the optimal solution for the total cost expressed in Eq. (9) is $x^* = (x_1^*, x_2^*, \ldots, x_N^*)$. Theorem 1 implies that to minimize the aggregate cost of the supply chain, each retailer i should be assigned an EOC period x_i^* as either zero or the manufacturers' lead time $L_0 + 1$. Notice that the aggregate cost provides a benchmark for the supply chain coordination. This theorem examines the ordering patterns of the retailers in the fully coordinated supply chain.

Unfortunately, although we know that in the coordinated system each retailer should only order regularly or with $L_0 + 1$ EOC periods, it is quite difficult to determine which retailer should be assigned to 0 or $L_0 + 1$ due to the complex interactions of different retailers' demand features, cost parameters, and ordering lead times in the cost function of the whole supply chain. Literally, there are totally 2^N combinations of possible solutions in a system with N retailers. One can select the optimal solution by evaluating each of these 2^N combinations and compare their corresponding costs using Eq. (9). To avoid using this time-consuming, brute force method, we develop two intuitive algorithms for solving the EOC assignment problem. We elaborate on the design of these algorithms below.

2.5. The Independent Algorithm and Greedy Algorithm

An intuitive approach to minimizing Eq. (9) is to treat the N retailers independently and solve N single-retailer problems separately. We call this approach *Independent Algorithm*. That is, for any retailer k, we solve the following problem:

$$\operatorname*{Min}_{x_k} \quad SC_k(x_k) = r_0\sqrt{\left(\frac{\sigma_k}{1-\rho_k}\right)^2 \sum_{j=L_k+x_k+2}^{L_k+L_0+2} (1-\rho_k^j)^2} + r_k\frac{\sigma_k}{1-\rho_k} \times \sqrt{\sum_{j=1}^{L_k+x_k+1} (1-\rho_k^j)^2}. \tag{10}$$

We call the solution generated from this approach the *independent solution* (IS), and denote the IS solution as $x^{IS} = (x_1^{IS}, x_2^{IS}, \ldots, x_N^{IS})$. Please note x_k^{IS} only takes the value of either 0 or $L_0 + 1$.

The relationship between IS solution $x^{IS} = (x_1^{IS}, x_2^{IS}, \ldots, x_N^{IS})$ and the optimal solution $x^* = (x_1^*, x_2^*, \ldots, x_N^*)$ is listed in Proposition 1.

Proposition 1. *For any retailer k, if $x_k^{IS} = 0$, then $x_k^* = 0$.*

Proposition 1 reveals that if retailer k does not use EOC in the single-retailer case, then she will not use EOC in the multiple retailer case. However, the reverse is generally not true: as shown in the following example, if a retailer does use EOC in the single-retailer case, she may not use EOC in the multiple retailer case. Although this independent algorithm does not generally provide the optimal solution to the original N-retailer EOC problem, it is helpful to refine the searching range. According to Proposition 1, we only need to focus on those retailers whose independent solution x_k^{IS} is positive (i.e., whose single-retailer solution adopts EOC).

Another intuitive way to minimizing Eq. (9) is to use a *Greedy Algorithm* which searches for the best EOC assignments step-by-step. We first solve the problem of minimizing Eq. (9) with respect to x_1 for retailer 1 to determine the optimal EOC period $x_1 = x_1^{GS}$ by assuming that all the other retailers do not use EOC (i.e., by fixing $x_i = 0$ for $i > 1$). We then solve the problem for retailer 2 to determine $x_2 = x_2^{GS}$ by fixing $x_1 = x_1^{GS}$, and $x_i = 0$ for $i > 2$. Whenever we obtain the values of x_i^{GS} for $i = 1,2,\ldots,k-1$, we fix them when we solve the problem for retailer k. The process is repeated for $i = 3,4,\ldots,N$ and stops after the single-retailer problem for $i = N$ is solved. We call the solution *Greedy Solution* (GS). Unfortunately, as shown by the following example, this algorithm generally does not provide the optimal solution, either.

Example 1. Consider the following example: There are $N = 2$ identical retailers with parameters as $r_1 = r_2 = 1.1$, $\sigma_1 = \sigma_2 = 2$, $\rho_1 = \rho_2 = 0.8$, $L_1 = L_2 = 4$. The parameters for the manufacturer are $r_0 = 1$, $L_0 = 10$ in one instance and $L_0 = 4$ in the other instance. For both instances, we can use Eq. (9) to calculate the total supply chain cost $SC(x)$ for different EOC periods $x = (x_1, x_2)$, where $x_i = 0$ or $L_0 + 1$ for $i = 1,2$, and the results are listed in Table 1. For the instance with $L_0 = 10$, Table 1a shows that the optimal solution to minimizing the total cost is $x^* = (0,0)$, i.e., neither retailers use EOC. This is the same as the GS solution $x^{GS} = (0,0)$. However, the IS solution is $x^{IS} = (L_0 + 1, L_0 + 1)$, i.e., both retailers use EOC. For the instance with $L_0 = 4$, Table 1b reveals that the optimal solution to minimizing the total cost is $x^* = (L_0 + 1, L_0 + 1)$, i.e., both retailers uses EOC. This is the same as the IS solution $x^{GS} = (L_0 + 1, L_0 + 1)$. However, the Greedy Solution is $x^{GS} = (0,0)$, i.e., neither retailer uses EOC. Since the retailers are identical in this instance, this example also shows that Greedy Algorithm cannot reach the optimal solution x^* even if we can re-label the retailer's index arbitrarily.

Table 1
Total supply chain cost for Example 1.

	(a) $L_0 = 10$		(b) $L_0 = 4$	
	$x_2 = 0$	$x_2 = L_0 + 1$	$x_2 = 0$	$x_2 = L_0 + 1$
$x_1 = 0$	66.1195	76.5328	50.2906	54.1863
$x_1 = L_0 + 1$	76.5328	69.5755	54.1863	47.2681

Since these two intuitive heuristics could not guarantee the finding of the optimal solution, we conduct a simulation experiment with 10,000 instances for each change of the number of retailers to test their performance on average. We find both the IS and GS are effective with certain percentage. Moreover, when the number of retailers increases, the possibility that the IS solution matches the optimal solution decreases, while the possibility that the GS solution matches the optimal solution increases. This observation shows that the IS solution is more suitable when N is small and the GS solution is more suitable when N is large. Essentially, the difference is due to the searching method. When N is large, the interaction among the retailers in the aggregate cost makes the independent searching less accurate, but make the step-by-step Greedy searching more meaningful. Example 2 describes this simulation experiment.

Example 2. We fixed $r_0 = 1$ and set the other parameters randomly: r_i is uniformly distributed on [1,1.5], σ_i is uniformly distributed on [1,5], ρ_i is uniformly distributed on [0.1,0.9], and L_i and L_0 are uniformly distributed on [1,10]. For each of $N = 2,3,4,5,6$, we generated 10,000 instances and calculated their corresponding optimal solutions x^*, the IS solutions x^{IS} and the GS solutions x^{GS}. The numerical results are summarized in Table 2. It reveals that when the number of retailers is small ($N = 2,3$), the optimal solution x^* equals to either the IS solution x^{IS} or the GS solutions x^{GS}. When the number of retailers increases, the percentage of the instances that the optimal solution x^* will be neither the IS solution x^{IS} nor the GS solutions x^{GS} also increases. However, even for $N = 6$, this percentage is only about 0.53%. These observations show that there is a high probability that the optimal solution x^* can be found in either the IS solution x^{IS} or the GS solutions x^{GS}. Examination of Table 2 also reveals that when the number of retailers increases, the possibility that the IS solution matches the optimal solution decreases, while the possibility that the GS solution matches the optimal solution increases. This pattern shows that the IS solution is more suitable when N is small and the GS solution is more suitable when N is large.

2.6. Effects of supply chain parameters

For the supply chain with a single manufacturer and a single retailer, Zhao et al. (2007) found that EOC has more possibility of reducing the total cost of the chain when (a) the inventory item receives less value-added activities at the retailer site; (b) the manufacturing lead time is short; (c) demand correlation over time is positive but weak; or (d) the delivery lead time is long. Basically, this is also true for the multiple retailer case according to Proposition 1. Besides, in the single-retailer case, though demand variation affects the absolute costs for all members in the supply chain, it does not affect the relative percentage cost savings due to the use of EOC. However, in the multiple retailer case, demand variation will also affect whether EOC is beneficial for the supply chain. Thus the interactions between the manufacturer and all the retailers become more complicated than the single-retailer case. This observation partially explains why it is more difficult to determine the optimal EOC policy in a multi-retailer case.

Moreover, the following Proposition 2 provides the conditions of the cost parameters when EOC should or should not be used. It reveals that for a retailer, only when her demand variation

Table 2
Numerical results for IS and GS.

No. of retailers (N)	Percentage of instances with $x^* = x^{IS} = x^{GS}$	Percentage of instances with $x^* = x^{IS}$ but $x^* \neq x^{GS}$	Percentage of instances with $x^* = x^{GS}$ but $x^* \neq x^{IS}$	Percentage of instances with $x^* \neq x^{IS}$ and $x^* \neq x^{GS}$
2	26.58	54.79	18.63	0
3	4.02	49.91	46.07	0
4	0.40	30.90	68.46	0.24
5	0.07	18.83	80.59	0.51
6	0	11.30	88.17	0.53

exceeds a threshold it is worthwhile for her to consider using EOC in the optimal solution. The exact value of the threshold depends on the parameters of the other retailers in the supply chain.

Proposition 2. *For any retailer k,*

(1) *EOC should never be used in the optimal solution, if*

$$\frac{r_0}{r_k} < \left(\sqrt{\sum_{j=1}^{L_k+L_0+2}(1-\rho_k^j)^2} - \sqrt{\sum_{j=1}^{L_k+1}(1-\rho_k^j)^2}\right) \Big/ \sqrt{\sum_{j=L_k+2}^{L_k+L_0+2}(1-\rho_k^j)^2}. \tag{11}$$

(2) *EOC should be used only when σ_k is bigger than certain threshold $\bar{\sigma}_k$, and*

$$\frac{r_0}{r_k} > \left(\sqrt{\sum_{j=1}^{L_k+L_0+2}(1-\rho_k^j)^2} - \sqrt{\sum_{j=1}^{L_k+1}(1-\rho_k^j)^2}\right) \Big/ \sqrt{\sum_{j=L_k+2}^{L_k+L_0+2}(1-\rho_k^j)^2}. \tag{12}$$

The first part of this proposition provides the requirement condition of the EOC for the cost parameters. Please note r_i is an increasing function of h_i and p_i ($i = 0, 1, \ldots, N$). Obviously, if r_0 is too small, or r_k is too big, which means the cost (holding cost h_k and shortage cost p_k) of the retailer k is too high and the cost (holding cost h_0 and shortage cost p_0) of the manufacturer is too low, No EOC should be used due to the inefficiency of transferring uncertainty to the retailer k. The rationale of the second part of the proposition is based on the cost tradeoff when a supply chain adopts EOC. On the one hand, adopting EOC adds uncertainty to retailer k and increases her cost; on the other hand, EOC reduces uncertainty of the manufacturer and decreases his cost. Due to the concavity of the cost gap function with respect to σ_k (as shown in the proof of Proposition 2), only when demand uncertainty σ_k is big enough, the cost reduction to the manufacturer exceeds the cost increase to the retailer, and hence EOC creates net savings for the supply chain.

With special selection of the parameters as shown in the following proposition, the optimal solution of the coordinated system can be easily found.

Proposition 3. *If* $\sum_{j=1}^{L_i+1}(1-\rho_i^j)^2\left(\frac{\sigma_i}{1-\rho_i}\right)^2$ *and* $\sum_{j=1}^{L_i+L_0+2}(1-\rho_i^j)^2\left(\frac{\sigma_i}{1-\rho_i}\right)^2$ *do not depend on retailer i, i.e., they are constants for each retailer i, then for any retailers j and k with $r_j < r_k$, retailer j uses EOC whenever retailer k uses EOC.*

Under the given conditions of this proposition, the optimal result can be found by ranking the retailers in the increasing order of the cost structure r_i, and by comparing the aggregate cost with the first i retailers using EOC. There are at most $N+1$ times of the comparisons. Essentially, this proposition provides the general condition under which the intervening effect among the retailers will be eliminated. For each retailer, only the cost tradeoff when shifting the demand uncertainty between the retailer and the manufacturer matters. It can also be easily shown that in this special case, the Greedy Algorithm can provide the optimal solution if we rank the retailers in the increasing order of the cost structure r_i.

3. Game-theoretic analysis for EOC-based price-discount scheme

The two algorithms developed in Section 2 provide high possibility to find the optimal solution that coordinates the supply chain with N retailers. However, without properly aligning the incentives of various members, it is still a difficult task for the decentralized supply chain to implement the solution and share the benefits of coordination. Although Zhao et al. (2007) propose a rebate scheme to address such an issue in the single-retailer case, their simple transfer of funds from the manufacturer to the retailer does not work reasonably for our multiple retailer setting. Their rebate presupposes that the manufacturer is the dominant player and hence can force the retailer to make an early commitment. Without explicit contracts, it is almost impossible for such a scheme to work in real industrial settings. For example, it is difficult for the manufacturer to reinforce compliance, especially when there are multiple retailers interacting with the manufacturer.

In this section, we develop an EOC-based wholesale price-discount scheme that allows the supply chain partners to divide the net savings resulting from EOC. Having committed to this pricing policy, the manufacturer can let the retailers decide what their EOC periods should be. Because the retailers now can voluntarily adopt EOC, instead of being forced to do it, the discount scheme is easier to implement in a decentralized supply chain in order to achieve coordination.

3.1. Wholesale price-discount scheme based on EOC

In the previous section, we presented the ordering decisions and associated cost evaluations for the supply chain members. In the ideal situation, the manufacturer would like to sign a pricing scheme with each individual retailer separately to reach complete supply chain coordination. However, such a pricing scheme discriminates among the retailers, and may be impeded from implementation according to the Roberson and Patman Act (Jeuland and Shugan, 1983; Ingene and Parry, 1995). In this section, instead, we conduct the analysis when the manufacturer uses a uniform EOC-based price discount to induce the retailers to practice EOC.

Theorem 1 in Section 2.4 motivates us to consider a simple price-discount scheme in which the manufacturer provides either no discount (to the retailers who do not order in advance), or full discount (to the retailers order as earlier as possible). Specifically, whenever a retailer places her orders x periods earlier, the manufacturer's wholesale per-unit price $w(x)$ takes the following form:

$$w(x) = \begin{cases} 1 & \text{if } x = 0, \\ 1-\alpha & \text{if } x = L_0 + 1. \end{cases} \tag{13}$$

That is, normalizing the wholesale price without EOC to be equal to 1, the wholesale price with EOC period being $L_0 + 1$ is assumed to be $1 - \alpha$ ($0 \leqslant \alpha \leqslant 1$).

The sequence of action is as follows: First, the manufacturer announces the pricing, basically the discount value α, to all the retailers. Observing this scheme, the retailers then determine whether to adopt EOC or not. Under this sequence, the decisions can be analyzed as a classical two-stage Stackelberg game. Notice that in a decentralized system each player, regardless of the manufacturer or any retailer, will only focuses on his/her own cost. However, by following the price-discount scheme, these individual incentives can be aligned with the optimal results for the whole supply chain (at least partially). Moreover, as we discussed in the following sections, the price-discount scheme is actually effective for wide range of parameters.

3.2. Retailers' decisions

From Eq. (1), it is easy to know that the long-run average demand per period at retailer i is

$$\overline{d_i} = \frac{d_i}{1-\rho_i}. \tag{14}$$

Thus the average purchasing cost at retailer i is

$$w(x_i)\overline{d_i} = \begin{cases} \frac{d_i}{1-\rho_i} & \text{if } x_i = 0, \\ \frac{(1-\alpha)d_i}{1-\rho_i} = \frac{d_i}{1-\rho_i} - \frac{\alpha d_i}{1-\rho_i} & \text{if } x_i = L_0+1. \end{cases} \tag{15}$$

Therefore, summarizing the average holding and shortage cost expressed in Eq. (4) and the average purchasing cost in Eq. (15) together (but excluding the part $\frac{d_i}{1-\rho_i}$, which is a constant for retailer i), we can express the total average cost for retailer i as

$$C_i(x_i) = c_i(x_i) + w(x_i)\overline{d_i} = \begin{cases} r_i \frac{\sigma_i}{1-\rho_i}\sqrt{\sum_{j=1}^{L_i+1}(1-\rho_i^j)^2} \\ \text{if } x_i = 0, \\ r_i \frac{\sigma_i}{1-\rho_i}\sqrt{\sum_{j=1}^{L_i+L_0+2}(1-\rho_i^j)^2} - \frac{\alpha d_i}{1-\rho_i} \\ \text{if } x_i = L_0+1. \end{cases} \tag{16}$$

The retailer will select the value for x_i to minimize its average cost in Eq. (16). That is to say, if $C_i(0) > C_i(L_0 + 1)$, i.e.,

$$r_i\frac{\sigma_i}{1-\rho_i}\sqrt{\sum_{j=1}^{L_i+1}(1-\rho_i^j)^2} > r_i\frac{\sigma_i}{1-\rho_i}\sqrt{\sum_{j=1}^{L_i+L_0+2}(1-\rho_i^j)^2} - \frac{\alpha d_i}{1-\rho_i} \tag{17}$$

or equivalently,

$$\alpha > \alpha_i \equiv r_i\frac{\sigma_i}{d_i}\left(\sqrt{\sum_{j=1}^{L_i+L_0+2}(1-\rho_i^j)^2} - \sqrt{\sum_{j=1}^{L_i+1}(1-\rho_i^j)^2}\right), \tag{18}$$

then retailer i will select to use EOC policy ($\bar{x}_i = L_0 + 1$). Otherwise, the retailer will not order in advance ($\bar{x}_i = 0$).

Notice that the threshold α_i, i.e., the right-hand side of inequality (18), depends on manufacturer's lead time, and the demand, cost and lead time parameters associated with retailer i. When retailer i observes the discount value of α provided by the manufacturer, the optimal response of the retailer ($\bar{x}_i$) can be easily determined. In fact, retailer i just needs to calculate her corresponding value of α_i, and compares it with the value of α. If $\alpha > \alpha_i$, then she will select $\bar{x}_i = L_0 + 1$; If $\alpha < \alpha_i$, $\bar{x}_i = 0$. For the special case of $\alpha = \alpha_i$, she will be indifferent between $\bar{x}_i = L_0 + 1$ and $\bar{x}_i = 0$. In the following discussions, without loss of generality, we assume that she will select $\bar{x}_i = L_0 + 1$ when $\alpha = \alpha_i$. Besides, if $\alpha_i > 1$, then inequality (18) will never hold, and retailer i will choose $\bar{x}_i = 0$. For this situation, we can redefine $\alpha_i = 1$ so that all the values of α_i are between 0 and 1.

3.3. Manufacturer's decision

Since the manufacturer can anticipate each retailer's response to his pricing scheme, he will provide the discount to minimize his total average cost per time period. In convenience of expositions, assume that the N retailers are indexed in the increasing order of their corresponding values of α_i, i.e.,

$$0 = \alpha_0 < \alpha_1 < \alpha_2 < \cdots < \alpha_N < \alpha_{N+1} = 1. \tag{19}$$

For a given discount $\alpha \in [\alpha_k, \alpha_{k+1})$ ($k = 0, 1, \ldots, N$), the first k retailers will place their orders in advance (i.e., $\bar{x}_i = L_0 + 1$ for $i = 1, 2, \ldots, k$), and the others will not (i.e., $\bar{x}_i = 0$ for $i = k+1, k+2, \ldots, N$). Considering the average holding and shortage cost expressed in Eq. (8) and the average discount given to the retailers, we can express the total average cost for the manufacturer as

$$\begin{aligned} C_0(\alpha) &= c_0(x) + \alpha \sum_{\substack{i=1 \\ x_i^*=L_0+1}}^{N} \overline{d_i} \\ &= r_0\sqrt{\sum_{i=k+1}^{N}\left(\frac{\sigma_i}{1-\rho_i}\right)^2 \sum_{j=L_i+2}^{L_i+L_0+2}(1-\rho_i^j)^2} + \alpha\sum_{i=1}^{k}\frac{d_i}{1-\rho_i}. \end{aligned} \tag{20}$$

Therefore, the manufacturer's total cost is a piecewise linear function of α with $(N+1)$ pieces, with each piece corresponding to a range of discount factor $\alpha \in [\alpha_k, \alpha_{k+1})$ ($k = 0, 1, \ldots, N$), as shown in Fig. 1. Furthermore, Eq. (20) implies that in each piece, the manufacturer's expected total cost is non-decreasing with respect to α. In order to minimize the total cost, the manufacturer simply needs to evaluate the values of $C_0(\alpha_k)$ ($k = 0, 1, \ldots, N$) expressed in Eq. (20) and select the optimal discount $\bar{\alpha} = \alpha_{\bar{k}}$ such that

$$\bar{k} = \arg\min_{k=0,1,\ldots,N} C_0(\alpha_k). \tag{21}$$

From the above discussions, the EOC-based price-discount scheme is easy to implement in practice. The manufacturer provides unit discount $\bar{\alpha} = \alpha_{\bar{k}}$ to the retailers who place their orders $L_0 + 1$ periods in advance, and the first $\bar{k}$ retailers will eventually practice the EOC policy.

3.4. Numerical examples and special cases

Although the price-discount scheme is easy to implement and can induce the retailers to practice EOC, a complete coordination may not be reached as it depends on the demand parameters and cost features of each retailer. In this section, we give several numerical examples that demonstrate the cost savings by EOC and test the efficiency of the price-discount scheme in reducing supply chain cost. After that, we provide a special case with certain demand parameters, under which the price-discount scheme can completely coordinate the supply chain cost.

In the following example, we illustrate the efficiency of the discount scheme with the change of retailer's lead time. We provide two numerical examples when coordination can or cannot be

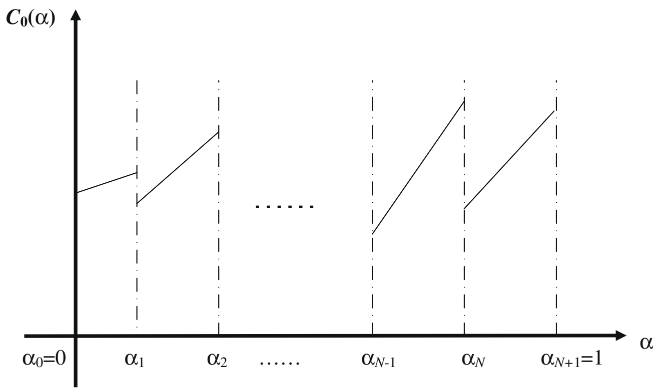

Fig. 1. The landscape of $C_0(\alpha)$ as a function of α.

Table 3
Parameters for Example 3.

	Manufacturer	Retailer i				
i	0	1	2	3	4	5
r_i	1	1.02	1.02	1.02	1.02	1.02
σ_i	N/A	0.5	0.8	1	0.5	1
ρ_i	N/A	0.5	0.4	0.5	0.6	0.3
d_i	N/A	5	8	10	5	10
L_i	3	18	15	15	15	9 or 3

reached. The numerical result show that even in the case when complete coordination cannot be reached (instance with $L_5 = 3$), the discount scheme still generates a good percentage of cost saving for the whole supply chain.

Example 3. Change of lead time. Assume there are $N = 5$ retailers with parameters given in Table 3. We compare two instances where the only difference in parameters is the lead time of Retailer 5, with $L_5 = 9$ in one instance and $L_5 = 3$ in the other instance.

Table 4 displays the corresponding costs for the manufacturer and each retailer under three supply chain policies: the No EOC case, the price-discount case with the discount factor $\bar{\alpha}$, and the complete coordination case, respectively. (The optimal solutions of the complete coordinated case were obtained by the brute force method discussed in Section 2.) It is shown that in both instances the supply chain cost is minimized when all retailers used EOC. Moreover, with the numerical results, it can be easily derived that the relative total cost saving of EOC compared with the No EOC case is 8.5% in the first instance ($L_5 = 9$) and 7.6% in the second instance ($L_5 = 3$). For the first instance, the discount scheme can achieve the complete coordination for the supply chain, with all the retailers using EOC policy. For the other instance, the discount scheme cannot achieve the complete coordination for the supply chain. Nevertheless, the total supply chain cost is indeed decreased compared with the No EOC case, after four retailers are induced to use EOC.

Next, we use the following example to illustrate the efficiency and effectiveness of the discount scheme when retailers have different cost structure. We also provide two different cases when the discount scheme can and cannot reach the complete supply chain coordination. Similar to the previous example, even when the complete coordination cannot be reached, there is still a big saving for the whole supply chain compared with No EOC case.

Example 4. Change of cost structure. In this example, we use five retailers with different parameters given in Table 5. The two comparing instances differ from each other in the cost structure of the fifth retailer in this example, with one instance $r_5 = 1.5$, and the other instance $r_5 = 1.1$.

The corresponding results of the three supply chain policies are displayed in Table 6. Different from Example 3, the optimal supply

Table 5
Parameters for Example 4.

	Manufacturer	Retailer i				
i	0	1	2	3	4	5
r_i	1	1.1	1.1	1.1	1.5	1.5 or 1.1
σ_i	N/A	1.5	1.5	1.5	3.0	3.0
ρ_i	N/A	0.6	0.7	0.8	0.3	0.2
d_i	N/A	5	5	5	10	10
L_i	3	15	15	15	2	2

chain cost for the instance when $r_5 = 1.5$ is reached when the first three retailers use EOC, rather than all the retailers use EOC. Again, the price-discount scheme can either reach the optimal result (as shown in the instance $r_5 = 1.5$), or decreases the supply chain cost compared with "No EOC" case (as shown in the instance $r_5 = 1.1$, where total supply chain cost is decreased from 99.2581 to 97.8401, with the optimal cost equal to 96.827). However, under the discount scheme, only the first three retailers use EOC in the instance $r_5 = 1.1$. In this specific instance, the relative effect of coordination due to the discount scheme measured by the supply chain cost savings is (99.2581 − 97.8401)/(99.2581 − 96.8270) = 58%.

In order to evaluate the discount scheme's performance under more generalized situations, we conducted the following numerical simulation experiment, in which we provide some meaningful numerical ranges for all the parameters and randomly generate 10,000 instances. The simulation experiment verifies that the performance of the price-discount scheme is no worse than that of the no discount scheme, and that the upper bound of the price-discount scheme is the supply chain cost under complete coordination. Moreover, the simulations show that the average cost saving of the price-discount scheme is 52% of the complete coordination.

Example 5. Simulation with 10,000 numerical instances. We fixed $N = 5$ and $r_0 = 1$, and set the other parameters randomly: r_i is uniformly distributed on [1, 1.5], σ_i is uniformly distributed on [1, 5], ρ_i is uniformly distributed on [0.1, 0.9], L_i is uniformly distributed on [1, 20], L_0 is uniformly distributed on [1, 5], and σ_i/d_i is uniformly distributed on [0.1, 0.3]. We generated 10,000 instances and calculated their corresponding supply chain costs under three policies: the No EOC case (SC^N), the price-discount case (SC^D), and the complete coordination case (SC^*), respectively. The simulation results are summarized in Table 7. Note that according to their cost implications, we should have $SC^* \leqslant SC^D \leqslant SC^N$. Both of columns 1 and 2 represent the instances that the discount scheme can lead to the minimal supply chain cost, which is accounted for 33.41% of all the 10,000 instances. For the rest of the instances, the discount scheme cannot lead to the minimal supply chain cost. In general, the average efficiency of the discount scheme, defined as the cost reduction over the No EOC policy, $(SC^N - SC^D)/(SC^N - SC^*)$, is more than 52%.

Table 4
Numerical results for Example 3.

L_5	Policy	Optimal discount factor $\bar{\alpha}$	Supply chain's cost (SC)	Manufacturer's cost (C_0)	Retailer's cost C_i ($i = 1,2,3,4,5$)				
					C_1	C_2	C_3	C_4	C_5
9	No EOC	0	32.7813	6.4438	4.2466	5.2421	7.7233	4.6956	4.4298
	Price discount	0.0611	29.9974	4.2834	4.1003[a]	5.0913[a]	7.5130[a]	4.5796[a]	4.4298[a]
	Complete coordination	–	29.9974	0	4.7112[a]	5.9058[a]	8.7348[a]	5.3432[a]	5.3024[a]
3	No EOC	0	30.9768	6.4428	4.2466	5.2421	7.7233	4.6956	2.6264
	Price discount	0.0518	30.1760	5.7474	4.1931[a]	5.2150[a]	7.6986[a]	4.6956[a]	2.6264
	Complete coordination	–	28.6162	0	4.7112[a]	5.9058[a]	8.7348[a]	5.3432[a]	3.9213[a]

[a] The relevant retailer uses EOC in the case.

Table 6
Numerical results for Example 4.

r_5	Policy	Optimal discount factor $\bar{\alpha}$	Supply chain's cost (SC)	Manufacturer's cost (C_0)	Retailer's cost C_i ($i = 1,2,3,4,5$)				
					C_1	C_2	C_3	C_4	C_5
1.5	No EOC	0	101.6513	22.4227	15.1917	19.2968	26.0908	9.6746	8.9748
	Price discount	0.0611	100.2334	21.4088	14.9700[a]	19.1145[a]	26.0908[a]	9.6746	8.9748
	Complete coordination	N/A	100.2334	11.3685	17.2870[a]	22.2038[a]	30.7248[a]	9.6746	8.9748
1.1	No EOC	0	99.2581	22.4227	15.1917	19.2968	26.0908	9.6746	6.5815
	Price discount	0.0518	97.8401	21.4088	14.9700[a]	19.1145[a]	26.0908[a]	9.6746	6.5815
	Complete coordination	N/A	96.8270	0	17.2870[a]	22.2038[a]	30.7248[a]	16.0611[a]	10.5504[a]

[a] The relevant retailer uses EOC in the case.

Table 7
Numerical results for the simulation study in Example 5.

Percentage of instances with $SC^* = SC^D = SC^N$	Percentage of instances with $SC^* = SC^D < SC^N$	Percentage of instances with $SC^* < SC^D \leqslant SC^N$	Average efficiency of the discount scheme (%): $(SC^N - SC^D)/(SC^N - SC^*)$
17.48	15.93	66.59	52.68

The above examples 3–5 illustrate that even if a complete coordination may not be reached, the total cost of the supply chain can still be improved by using the EOC-based price-discount scheme, in comparison to the No EOC case. Moreover, as stated in the following proposition when the demands of the N retailers have certain characteristics, the price-discount scheme can fully coordinate the total supply chain.

Proposition 4. *Suppose all the retailers face the same type of demand and have the same ordering lead time (i.e., $d_i = d$, $\sigma_i = \sigma$, $\rho_i = \rho$, and $L_i = L$ for all retailer i), and the retailers are indexed as $r_1 < r_2 < \cdots < r_N$. If the cost parameter $r_i/(\sqrt{N-i+1} - \sqrt{N-i})$ is increasing in i ($i = 1,2,\ldots,N$), and there is a retailer k ($1 < k < N$) such that*

$$\frac{kr_k - (k-1)r_{k-1}}{\sqrt{N-k+1} - \sqrt{N-k}} \leqslant \frac{r_0\sqrt{\sum_{j=L+2}^{L+L_0+2}(1-\rho^j)^2}}{\sqrt{\sum_{j=1}^{L+L_0+2}(1-\rho^j)^2} - \sqrt{\sum_{j=1}^{L+1}(1-\rho^j)^2}} \leqslant \frac{r_{k+1}}{\sqrt{N-k} - \sqrt{N-k-1}} \quad (22)$$

and $\frac{kr_k - ir_i}{\sqrt{N-i} - \sqrt{N-k}}$ is increasing in i for all $i \neq k$, then both the supply chain optimal cost and the manufacturer's minimal cost under the price-discount scheme can be reached at the same time with the first k retailers using EOC and the others not using EOC, i.e., the price-discount scheme coordinates the total supply chain with the minimum total cost.

Intuitively, with the demand parameters provided as in Proposition 4, the complex N-retailer ordering problem with AR(1) demand pattern can be solved to optimality by the Greedy Algorithm. After rearrange the index of the retailers as the cost decreasing order, the condition (22) provides an enough gap between the cost structure of retailer k and retailer $k+1$ so that it is efficient for the first k retailers to use EOC due to their cost efficiency and for the later $N-k$ retailers to order regularly due to their less-efficiency. This proposition can be illustrated by the following numerical example.

Example 6. Full coordination with special parameters. Here we give an instance for Proposition 4 with $N = 5$, $r_0 = 1$, $L_0 = 1$, and $d_i = d$, $\sigma_i = \sigma$, $\rho_i = \rho = 0$, $L_i = L = 11$ for all retailer i. (Here d and σ can be any positive numbers.) The cost parameters for the retailers are as follows: $r_1 = 1.05$, $r_2 = 1.2$, $r_3 = 1.7$, $r_4 = 2.3$, $r_5 = 5.6$. Under this setting, the middle item in the condition (22) has a value of 5.095. From Table 8, we can easily see that under this situation, all the conditions of Proposition 4 are satisfied with $k = 2$. Therefore, the price-discount scheme coordinates the system with the minimal total cost (i.e., only the first two retailers use EOC).

Table 8
Numerical example for Proposition 4.

i	r_i	$\frac{r_i}{\sqrt{N-i+1}-\sqrt{N-i}}$	$\frac{kr_k-ir_i}{\sqrt{N-i}-\sqrt{N-k}}$
1	1.05	4.448	5.038
2	1.2	4.478	N/A
3	1.7	5.349	8.495
4	2.3	5.553	9.289
5	5.6	5.600	15.935

4. Conclusions

In this paper, we study the practice of EOC in a decentralized, two-level supply chain consisting of a single manufacturer and multiple independent retailers with AR(1) demands. We formulate the supply chain cost model and develop the property of its optimal solution. We then examine three solution approaches to solving the optimal solution for coordinating the supply chain. Furthermore, we develop a wholesale price-discount scheme that allows the supply chain partners to share the net savings resulting from EOC. This discount scheme can induce the retailers to voluntarily participate in EOC. Our simulation experiments show that in 33% of the 10,000 incidences tested, the discount scheme led to minimal supply chain cost. Although the EOC-based wholesale price-discount scheme cannot guarantee to reach an optimal solution, it still helps to reduce supply chain cost in comparison of using the No EOC policy. In addition, we also analyze the conditions where the supply chain can be coordinated through the discount scheme.

While this model provides insights into the implementation of EOC policy in practice, it has several limitations. First, although we provide a simple implementation scheme for the manufacturer to induce the retailers to practice EOC policy, this scheme may not fully coordinate the partners, and thus it is meaningful to conduct further research to evaluate the efficiency of the EOC-based discount scheme proposed in this paper. Second, it will be also helpful to study other incentive schemes, especially to find a simple scheme which can lead to the best performance of the total supply chain. Finally, in developing the model we assume that the demand follows a first order autoregressive process and that all parties know the demand parameters. It will be useful to extend the model to cases where the demand follows other distributions or there are uncertainties concerning the distribution parameters.

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Appendix A. Supplementary data

The proofs for Theorem 1 and Propositions 1–4. Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2008.12.027.

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