



Interface with Other Disciplines

Coordinating advertising and pricing in a manufacturer–retailer channel

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ABSTRACT

Cooperative advertising is a practice that a manufacturer pays retailers a portion of the local advertising cost in order to induce sales. Cooperative advertising plays a significant role in marketing programs of channel members. Nevertheless, most studies to date on cooperative advertising have assumed that the market demand is only influenced by advertising expenditures but not by retail price. This paper addresses channel coordination by seeking optimal cooperative advertising strategies and equilibrium pricing in a two-member distribution channel. We establish and compare two models: a non-cooperative, leader–follower game and a cooperative game. We develop propositions and insights from the comparison of these models. The cooperative model achieves better coordination by generating higher channel-wide profits than the non-cooperative model with these features: (a) the retailer price is lower to consumers; and (b) the advertising efforts are higher for all channel members. We identify the feasible solutions to a bargaining problem where the channel members can determine how to divide the extra profits.

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1. Introduction

Coordination between independent firms in a supply chain relationship has gained much attention recently. Without coordination, distribution channel members determine their own decision variables independently in order to maximize one's own profits. It is well documented in marketing and economics literature that uncoordinated decisions lead to “double marginalization”, which is one of the causes of channel inefficiency (Spengler, 1950; Tirole, 1989; Gerstner and Hess, 1995). Without coordination, channel members' individual profits may be inferior to what could be achieved if they work together to reach decisions that can expand the market so as to maximize the joint profits of all channel members. Many studies about channel coordination have been presented in the literature, addressing different dimensions of business decisions, including advertising, pricing, production, purchasing, and inventory management (Berger, 1972; Jeuland and Shugan, 1983; Eliashberg and Steinberg, 1987; Weng, 1995; Ingene and Parry, 2000). Nevertheless, studies simultaneously dealing with more than one aspect of coordination are complex and sparse. This paper is concerned with the conflicts and the coordination between pricing and advertising strategies for a manufacturer–retailer channel relationship.

A fundamental decision for supply chain coordination is pricing, which typically includes wholesale price and retail price. Pricing is a core theme in the marketing research literature on distribution

channels. Early works on pricing often study a bilateral monopoly (i.e., a single manufacturer sells through an exclusive but independent retailer) model and have identified vertical coordination through quantity-discounts (Jeuland and Shugan, 1983), two-part tariffs (Moorthy, 1987; Jeuland and Shugan, 1988a), implicit understanding (Shugan, 1985), and formation of conjectures (Jeuland and Shugan, 1988b). Complexity of pricing arises as the number of suppliers, buyers, and products increase. McGuire and Staelin (1983) find that product substitutability may substantially influence the equilibrium distribution structure. Lal and Staelin (1984) extends the concept of discount pricing to a group of buyers to minimize the joint buyer and seller ordering and holding costs. Choi (1991) examines a market where a common retailer sells the products of two competing manufacturers. Choi studies the effects of power structure, product differentiation, and cost differences on channel coordination and finds that many of the results depend heavily on the demand function employed. Building upon their continual works on channel pricing, Ingene and Parry (1995a,b, 1998, 2000) argue that when a manufacturer sells to multiple competing retailers, establishing a wholesale price policy for channel coordination is often undesirable relative to utilizing a non-coordinating, “sophisticated Stackelberg” two-part tariff. As a departure from offering incentives to retailers, Gerstner and Hess (1995) recommend that manufacturers use a “pull” price discount directly to price-sensitive consumers to achieve channel coordination.

Companies allocate advertising budgets with the ultimate goal of stimulating consumer purchases. A manufacturer's advertising tends to be spent at a national level for building the long-term image or “brand equity” for the company or for some of its major

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products. On the other hand, a retailer's advertising usually targets at inducing short-term purchase through local media (Young and Greyser, 1983; Houk, 1995). Vertical cooperative (co-op) advertising is an arrangement whereby a manufacturer pays for some or all of the costs, commonly referred as the "participation rate", of local advertising undertaken by a retailer for that manufacturer's products (Bergen and John, 1997). In the absence of co-op advertising, the retailer would typically advertise less than the level desired by the manufacturer. Thus, co-op advertising plays a significant role in the manufacturer–retailer channel relationship. Brennan (1988) reports that in the personal computer industry, IBM offers a 50–50 split of advertising costs with retailers while Apple Computer pays 75% of the media costs. Karray and Zaccour (2007) indicate that marketing research firms like the National Register Publishing collects more than 4000 co-op programs subsidized by manufacturers in 52 product classifications. In large sample empirical studies, both Dutta et al. (1995) and Nagler (2006) find that the most common participate rates were set at 50% and 100%. Nagler (2006) indicates that the total expenditures on cooperative advertising in 2000 were estimated at \$15 billion in the USA, nearly a four-fold increase in real terms in comparison to \$900 million in 1970. On the other hand, real total advertising expenditure grew by a factor less than three during the same period. The overall significance and growth trend of cooperative advertising suggest a need of more research for understanding its role and use in practice.

Studies of co-op advertising typically examine advertisement efforts in dimensions such as national level expenditures, local level expenditures, manufacturer participation rate, sales volume, brand and store substitutions, among others. These factors represent inherent interdependence and conflict between the interests of channel partners. Because of the nature of cooperative advertising, ample opportunities for channel coordination exist. Recently, game theoretical model has become a popular vehicle to analyzing co-op advertising. Some studies focus on analyzing a channel relationship through a static, single-period lens in order to explore the detailed interactions among the factors involved in co-op advertising (Dant and Berger, 1996; Bergen and John, 1997; Kim and Staelin, 1999; Karray and Zaccour, 2006, 2007). Marketing literature often assumes an asymmetric relationship where a powerful manufacturer tends to be the driving force for channel coordination. Motivated by observing the power shift from manufacturers to retailers in recent years, represented by the rise of Wal-Mart, Huang and Li (2001) and Huang et al. (2002) develop and compare two models to reflect different power structure and the corresponding ways of coordinating advertisement spending.

Another branch of game theoretical models on co-op advertising takes a long-term perspective by studying the dynamic, intra-channel relationships between channel members over certain time periods. Dynamic game theoretic models are typically based on a goodwill function associated with the brand image that is influenced through national and local advertising efforts (Nerlove and Arrow, 1962; Chintagunta and Jain, 1992; Jorgensen et al., 2000; Jorgensen et al., 2003). The brand goodwill functions used in these models often assume that short-term, local advertising has no effect on brand image. However, results from empirical studies are mixed and range from negative to positive effects (Jorgensen et al., 2000; Jorgensen and Zaccour, 2003b). These dynamic models ignore the manufacturer participation rate which is pivotal in co-op advertising practice. Another common problem of these dynamic models is that pricing is independent of advertising decisions (Jorgensen et al., 2003, p. 818). As a result, the equilibrium prices are constant over time and need to be obtained from a static game.

This research is motivated by the scarcity of models that simultaneously incorporate two main factors in channel coordination:

pricing policy and co-op advertising efforts at both the national and local levels. Both factors are significant determinants of market demand and hence profits earned by channel members. Nevertheless, we can only find a handful of studies that deal with these two factors together. Bergen and John (1997) consider wholesale price, retail price, participation rate, and intra-brand competition across multiple retailers in a static game. Their model focuses only on local advertising in order to study "the most prominent aspect of these (co-op advertising) plans" – the effect of manufacturer participation rate. Jorgensen and Zaccour (1999) and Jorgensen and Zaccour (2003a) model consumer demand as the multiplicative product of retail price and goodwill in a dynamic setting. They compare coordinated strategies and profits with uncoordinated ones and then discuss how a coordinated solution could be sustained over time. Karray and Zaccour (2006) found that co-op advertising can be an efficient counter-strategy for the manufacturer when a retailer offers a private label product that may affect the national brand. In their model, national advertising is not considered, and the manufacturer participates in local advertising only when both parties cooperate in advertising efforts. Yue et al. (2006) extend the co-op advertising model of Huang et al. (2002) by incorporating price elasticity to study the effect of direct manufacturer price discount on channel coordination. Their approach is unique in that a manufacturer can bypass the retailer and directly gives the consumer a price deduction from the suggested retail price, such as using a rebate or coupon. They recommend that coordination in local and national co-op advertising with a partnership scheme is the best solution.

In this study, we develop two models where consumer demand is determined by retail price and co-op advertising efforts by channel members. We confine our interest to the traditional setting of a bilateral monopoly model in which one manufacturer sells through one retailer. Focusing on static models allows us to develop analytical solutions and insights to key factors, including manufacturer wholesale price, retail price, the advertising expenditures by the channel members, and the manufacturer's participation rate. We find that the cooperative model achieves better channel coordination and generates higher channel-wide profits than the non-cooperative model with these features: (a) the retailer price is lower to consumers; and (b) the advertising efforts are higher for both channel members. We identify the feasible solutions to a bargaining problem where the channel members can determine how to share the additional profits.

The paper proceeds as follows: the next section presents the assumptions and the basic game-theoretic model structure. Then two models are discussed, one based on a non-cooperative game (the manufacturer as the leader and the retailer as the follower), and the other a cooperative game. The main results of these two models are analyzed and compared, followed by the discussion of the bargaining problem. Finally, the conclusion summarizes the findings and proposes directions for future research. Derivation of key results and proof of propositions are relegated to the Appendix.

2. Assumptions and the basic market structure

We consider a single-manufacturer-single-retailer channel in which the retailer sells only the manufacturer's brand within the product class. Decision variables for the channel members are their advertising expenditures, their prices (manufacturer's wholesale price and retailer's retail price) and the manufacturer participation rate. The variables a and A denote the retailer's local advertising expenditure and the manufacturer's national advertising expenditure, respectively. The consumer demand $V(p, a, A)$ depends on the retail price p and the advertising levels a and A as

$$V(p, a, A) = g(p)h(a, A), \tag{1}$$

where $g(p)$ reflects the impact of the retail price on the demand, and $h(a, A)$ reflects the impact of the advertising expenditures on the demand, also known as the sale response function. Using a multiplicative effect by price and advertising to model consumer demand was commonly seen in the literature (Kuehn, 1962; Thompson and Teng, 1984; Jorgensen and Zaccour, 1999, 2003; Yue et al., 2006). We further assume that $g(p)$ is linearly decreasing with respect to p , which is a well known demand function in the literature (i.e., Jeuland and Shugan, 1988; Weng, 1995). Specifically, we assume

$$g(p) = 1 - \beta p, \tag{2}$$

where β is a positive constant. Please note that the maximum value for $g(p)$ is normalized to be 1 for simplicity of the expressions. Besides, in order to ensure $g(p) > 0$, we need to restrict $p < 1/\beta$.

There is a substantial literature on the estimation of the sales response function with respect to advertising expenditures. Some studies do not distinguish the impacts on sales between local (retailer) and national (manufacturer) advertising expenditures (Berger, 1972; Little, 1979; Tull et al., 1986; Dant and Berger, 1996). We are in accord with the notion that both types of advertising efforts could influence sales and their effects should be assessed separately (Jorgensen et al., 2000; Huang et al., 2002). Thus, we model advertising effects on consumer demand as

$$h(a, A) = k_r \sqrt{a} + k_m \sqrt{A}, \tag{3}$$

where k_r, k_m are positive constants reflecting the efficacy of each type of advertising in generating sales. Eq. (3) captures both types of advertising effects which usually are not substitutes. The demand in (3) is an increasing and concave function with respect to a and A , and has the property that is consistent with the commonly observed “advertising saturation effect”, i.e., additional advertising spending generates continuously diminishing returns. After reviewing over 100 studies, Simon and Arndt (1980) conclude that diminishing returns characterize the shape of the advertising-sales response function. Similar approaches of relating demand and advertising expenditure were used in Kim and Staelin (1999) and Karray and Zaccour (2006). Furthermore, denote $k = k_m^2/k_r^2$, which will be used to simplify the expressions later in this paper. In the discussion below, k will be called the advertising ratio, reflecting the relative effectiveness of national versus local advertising in generating sales.

Combining (1)–(3), we have

$$V(p, a, A) = (1 - \beta p)(k_r \sqrt{a} + k_m \sqrt{A}). \tag{4}$$

We denote by t the manufacturer participation rate, the percentage that the manufacturer agrees to pay the retailer to subsidize the local advertising cost, and by w the manufacturer’s wholesale price to the retailer. Furthermore, the manufacturer’s unit production cost and the retailer’s unit handling cost incurred in addition to the purchasing cost are assumed to be constants, thus they can be normalized to zero for simplicity of the expressions.

The profits of the manufacturer, the retailer and the system are as follows, respectively:

$$\Pi_m = w(1 - \beta p)(k_r \sqrt{a} + k_m \sqrt{A}) - ta - A, \tag{5}$$

$$\Pi_r = (p - w)(1 - \beta p)(k_r \sqrt{a} + k_m \sqrt{A}) - (1 - t)a, \tag{6}$$

$$\Pi_{m+r} = p(1 - \beta p)(k_r \sqrt{a} + k_m \sqrt{A}) - a - A. \tag{7}$$

Remark. Throughout this paper, the subscript “m”, “r” and “m + r” means the parameters corresponding to the manufacturer, the retailer, and the whole system.

Please note that A, w and t are manufacturer’s decision variables, and a, p are retailer’s decision variables, where $0 < w < p < 1/\beta, 0 \leq t \leq 1$, and a, A may take any nonnegative real values.

3. The leader–follower relationship model

In this section, we model the decision process as a sequential, non-cooperative game, with the manufacturer as the leader and the retailer as the follower. The solution of this leader–follower game is called the Stackelberg equilibrium. In order to determine the Stackelberg equilibrium by backward induction, we first solve the retailer’s optimal problem when the manufacturer’s decision variables A, w and t are given:

$$\begin{aligned} \text{Max } \Pi_r &= (p - w)(1 - \beta p)(k_r \sqrt{a} + k_m \sqrt{A}) - (1 - t)a \\ \text{s.t. } &0 < p < 1/\beta \text{ and } a > 0. \end{aligned} \tag{8}$$

Since Π_r is a concave function with respect to a and p , we can solve the two first order equations $\partial \Pi_r / \partial a = 0$ and $\partial \Pi_r / \partial p = 0$ to get the optimal values:

$$p = (1 + \beta w) / 2\beta, \tag{9}$$

$$a = k_r^2 (1 - \beta w)^4 / 64\beta^2 (1 - t)^2. \tag{10}$$

It is interesting to note from (9) that the retailer’s best response for setting a retail price (p) is a linearly increasing function of the manufacturer’s wholesale price (w), but depends on neither the manufacturer’s advertising expenditure (A) nor the participation rate (t) for subsidizing the retailer’s advertising. Examination of (10) reveals that the retailer’s best response for local advertising level (a) decreases as the manufacturer’s wholesale price (w) increases, and increases as the manufacturer’s participation rate (t) increases. The optimal local advertising level does not depend on the manufacturer’s advertising expenditure (A).

Next, the optimal values of A, w and t are determined by maximizing the manufacturer’s optimal problem:

$$\begin{aligned} \text{Max } \Pi_m &= w(1 - \beta p)(k_r \sqrt{a} + k_m \sqrt{A}) - ta - A \\ \text{s.t. } &0 \leq t \leq 1, 0 < w < 1/\beta \text{ and } A > 0, \end{aligned} \tag{11}$$

where p and a are determined by Eqs. (9) and (10), respectively.

Solving this decision problem, we obtain the Stackelberg equilibrium results (w^*, A^*, t^*, p^*, a^*) listed in Table 1 and have the following important observations (see the Appendix for the proof).

Proposition 1. *The Stackelberg game, where the manufacturer as the leader and the retailer as the follower, has a unique equilibrium (w^*, A^*, t^*, p^*, a^*) with these properties:*

- (i) $1/3\beta < w^* < 1/2\beta$ and $\frac{\partial w^*}{\partial k} > 0$.
- (ii) $1/3 < t^* < 3/5$ and $\frac{\partial t^*}{\partial k} > 0$.
- (iii) $2/3\beta < p^* < 3/4\beta$ and $\frac{\partial p^*}{\partial k} > 0$.
- (iv) $\frac{\partial A^*}{\partial k_m} > 0, \frac{\partial a^*}{\partial k_m} < 0, \frac{\partial A^*}{\partial k_r} < 0, \frac{\partial a^*}{\partial k_r} > 0$.

Part (i) of Proposition 1 shows that the manufacturer’s wholesale price (w^*) is within the range $(1/3\beta, 1/2\beta)$, and part (iii) of Proposition 1 shows that the retailer’s selling price (p^*) is within the range $(2/3\beta, 3/4\beta)$. Notice that $1/\beta$ is the largest possible value for both the wholesale and retail prices. Part (i) means that the wholesale price (w^*) is always greater than one-third but less than one-half of the highest possible price. Likewise, for the retail price, it is always between two-third and three-fourth of the highest possible price. Part (ii) of Proposition 1 indicates that the manufacturer should share at least one-third of the local advertising expenditure, and at most 60% of the local advertising expenditure. Parts (i)–(iii) of Proposition 1 also reveal that the wholesale price

Table 1
Comparison of the results for the two models

Stackelberg game	Partnership game
$w^* = \frac{4k + \sqrt{16k^2 + 16k + 9}}{\beta(9 + 16k)}$	$\frac{k_r^2 t + k_m^2 + 64\beta \Pi_m^*}{4(k_r^2 + k_m^2)} \leq w \leq \frac{k_r^2 t + k_m^2 + 2k_m^2 - 64\beta^2 \Pi_r^*}{4(k_r^2 + k_m^2)}$
$A^* = k_m^2 [w^* (1 - \beta w^*)]^2 / 16$	$\bar{A} = k_m^2 / 64\beta^2$
$t^* = (5\beta w^* - 1) / (3\beta w^* + 1)$	$0 \leq t \leq 1$
$p^* = (1 + \beta w^*) / 2\beta$	$\bar{p} = 1 / 2\beta$
$a^* = k_r^2 [(3\beta w^* + 1)(1 - \beta w^*)]^2 / 256\beta^2$	$\bar{a} = k_r^2 / 64\beta^2$
$\Pi_{m+r}^* = \frac{(1 - \beta w^*)^2}{256\beta^2} [3k_r^2 (\beta w^*)^2 + (10k_r^2 + 16k_m^2)\beta w^* + 3k_r^2]$	$\bar{\Pi}_{m+r} = (k_r^2 + k_m^2) / 64\beta^2$
$\Pi_m^* = \frac{(1 - \beta w^*)^2}{256\beta^2} [(9k_r^2 + 16k_m^2)(\beta w^*)^2 + 6k_r^2 \beta w^* + k_r^2]$	$\bar{\Pi}_m = [4(k_r^2 + k_m^2)\beta w - k_r^2 t - k_m^2] / 64\beta^2$
$\Pi_r^* = \frac{(1 - \beta w^*)^2}{128\beta^2} [-(3k_r^2 + 8k_m^2)(\beta w^*)^2 + (2k_r^2 + 8k_m^2)\beta w^* + k_r^2]$	$\bar{\Pi}_r = [k_r^2 + k_r^2 t + 2k_m^2 - 4(k_r^2 + k_m^2)\beta w] / 64\beta^2$

(w^*), the participation ratio (t^*) and the retail price (p^*) all increase as the advertising ratio k increases. Since the advertising ratio k represents the impact of the national advertisement on the sales volume relative to the local advertisement, this is consistent with the intuition. Part (iv) of Proposition 1 means that the manufacturer's advertising level (A^*) increases with respect to the efficacy of the manufacturer's advertising efforts (k_m) and decreases with respect to the efficacy of the retailer's advertising efforts (k_r). Conversely, the retailer's advertising level (a^*) increases with respect to the efficacy of the retailer's advertising efforts (k_r) and decreases with respect to the efficacy manufacturer's advertising efforts (k_m). This is also consistent with the intuition.

From the expression of w^* in Table 1, the optimal wholesale price w^* depends only on β and the advertising ratio (k). It is easy to see that w^* is a decreasing function of β , and this is reasonable because β represents the sensitivity of the sales volume to the retail price as defined in Eq. (2). The more sensitive the sales volume to the retail price, the lower the wholesale price.

4. The cooperative relationship model

In this section, we focus on a cooperative game structure in which both the manufacturer and the retailer agree to make decisions that maximize the total channel profits (joint profit maximization).

The system profits are described by Eq. (7) and depend only on p , a and A . We hence have the following optimization problem:

$$\begin{aligned} \text{Max } & \Pi_{m+r} = p(1 - \beta p)(k_r \sqrt{a} + k_m \sqrt{A}) - a - A \\ \text{s.t. } & 0 < p < 1/\beta \text{ and } a, A > 0. \end{aligned} \tag{12}$$

This problem can easily be solved by equating the three partial derivatives to zero. Specifically, by taking $\partial \Pi_{m+r} / \partial p = 0$, $\partial \Pi_{m+r} / \partial a = 0$ and $\partial \Pi_{m+r} / \partial A = 0$, we have the unique solution ($\bar{p}, \bar{a}, \bar{A}$) expressed as

$$\bar{p} = \frac{1}{2\beta}, \tag{13}$$

$$\bar{a} = \frac{k_r^2}{64\beta^2}, \tag{14}$$

$$\bar{A} = \frac{k_m^2}{64\beta^2}. \tag{15}$$

These results reveal that in order to maximize the profit for the total channel chain, the retail price should be set at one-half of the maximum-possible value ($1/\beta$), and it is independent of the advertising ratio. Furthermore, the local and national advertising expenditures should be set at the values specified in Eqs. (14) and (15), which are dependent on the corresponding efficacy coefficient k_r and k_m .

Therefore, the optimal profits for the whole system can be calculated as

$$\bar{\Pi}_{m+r} = \frac{k_r^2 + k_m^2}{64\beta^2}. \tag{16}$$

Comparing the results of the two models, we have the following observations (see Appendix for the proof).

Proposition 2

- (i) $\bar{p} < \frac{3}{4} p^* < p^*$.
- (ii) $A^* < \frac{1}{4} \bar{A} < \bar{A}$, $a^* < \frac{4}{9} \bar{a} < \bar{a}$.
- (iii) $\Pi_{m+r}^* < \frac{20}{27} \bar{\Pi}_{m+r} < \bar{\Pi}_{m+r}$.

Proposition 2 basically shows that in a cooperative model higher channel profits are achieved by using a lower retail price and a higher level of advertising efforts by both channel partners to stimulate demand. Interestingly, we note that the same observations about retail price and advertising efforts were made in Jeuland and Shugan (1983, p. 267) as well as in Jorgensen and Zaccour (1999, p.119). However, the former study does not model advertising effect explicitly, while the latter employs a dynamic game model without studying manufacturing participating rate. Besides, Part (iii) of Proposition 2 reveals that in our specific model, moving to cooperation can improve the total channel profit by at least $7/20 = 35\%$, compared with the leader–follower game setting.

If p , a and A are respectively, equal to the unique solution ($\bar{p}, \bar{a}, \bar{A}$), then the channel profits (16) are maximized with t being free to take any value between 0 and 1 and w being free to take any value between 0 and $\bar{p} = 1/2\beta$. However, note that both the manufacturer's profits and the retailer's profits are dependent of t and w as shown in Table 1. Neither the manufacturer nor the retailer would be willing to maximize the channel profits through cooperation if their own profits are lower than those in a Stackelberg game. Therefore, a mechanism must exist to provide incentive for the channel members to cooperate and share the extra-profits. This issue is addressed in the next section as the bargaining problem.

5. The bargaining problem

The extra-profits accrued from the cooperative game relative to the Stackelberg game can be expressed as $\Delta \Pi_{m+r} = \bar{\Pi}_{m+r} - \Pi_{m+r}^*$, with Π_{m+r}^* being the channel profits under the Stackelberg game and $\bar{\Pi}_{m+r}$ being the channel profits under the partnership game. Part (iii) of Proposition 2 indicates that the extra-profits $\Delta \Pi_{m+r}$ are greater than zero. Now we discuss how such extra-profits should be jointly shared between the manufacturer and the retailer.

In order to ensure that both the manufacturer and the retailer are willing to participate in a cooperative rather than a leader–follower relationship, we face a bargaining problem over $0 \leq w \leq \bar{p} = 1/2\beta$ and $0 \leq t \leq 1$, subject to

$$\Delta\Pi_m = \bar{\Pi}_m - \Pi_m^* \geq 0 \quad \text{and} \quad \Delta\Pi_r = \bar{\Pi}_r - \Pi_r^* \geq 0, \quad (17)$$

where Π_m^* and Π_r^* are manufacturer’s and retailer’s profits, respectively, under the Stackelberg game, and $\bar{\Pi}_m$ and $\bar{\Pi}_r$ are the corresponding profits under the cooperative game. That is, $\Delta\Pi_m$ and $\Delta\Pi_r$ are the extra-profits that can be made by the manufacturer and the retailer, respectively. Obviously, $\Delta\Pi_m + \Delta\Pi_r = \Delta\Pi_{m+r}$.

As to the feasible region about the bargaining situation, we present the following observation (see Appendix for the proof):

Proposition 3. Both the manufacturer and the retailer would be willing to move to the cooperative relationship if and only if

$$\frac{k_r^2 t + k_m^2 + 64\beta\Pi_m^*}{4(k_r^2 + k_m^2)} \leq w \leq \frac{k_r^2 t + k_r^2 + 2k_m^2 - 64\beta^2\Pi_r^*}{4(k_r^2 + k_m^2)}$$

and $0 \leq t \leq 1$.

The feasible region is plotted in Fig. 1 with respect to (w, t) . All the points located in between the two boundary lines $\bar{\Pi}_m = \Pi_m^*$ and $\bar{\Pi}_r = \Pi_r^*$ are feasible solutions. The slope of these boundary lines is $4(1 + k_m^2/k_r^2)$. The closer a solution gets to $\bar{\Pi}_m = \Pi_m^*$, the bigger the manufacturer’s share $\Delta\Pi_m$ of the joint extra-profits $\Delta\Pi_{m+r}$ and the smaller the retailer’s share $\Delta\Pi_r$. All the points located on a line parallel to $\bar{\Pi}_m = \Pi_m^*$ (or $\bar{\Pi}_r = \Pi_r^*$) lead to the same profits for the manufacturer as $\bar{\Pi}_m = \Pi_m^* + \Delta\Pi_m$ (and for the retailer as $\bar{\Pi}_r = \Pi_r^* + \Delta\Pi_r$). As a result, in order to earn the same level of the extra-profits, the manufacturer can choose to simultaneously increase or decrease the values of (w, t) . This effect is reminiscent of the findings by Tull et al. (1986) who suggest that advertising and margin are closely related and advertising policy should be modified as pricing and margin is changed.

However, we cannot determine more precisely the values of (w, t) without any further information. One possible approach to solving the bargaining problem is the Nash bargaining model (Nash, 1950). The bargaining outcome is obtained by maximizing the product of individual marginal utilities over the feasible solution region. For example, assume the utility functions for the manufacturer and the retailer are given by $u_m(\Delta\Pi_m) = (\Delta\Pi_m)^{\lambda_m}$ and

$u_r(\Delta\Pi_r) = (\Delta\Pi_r)^{\lambda_r}$ with some positive constant λ_m and λ_r , respectively. Usually, λ_m and λ_r can be explained as the risk attitude of the manufacturer and the retailer when the bargaining game has a risk of breakdown, with a larger value meaning more risk-seeking. Then the Nash bargaining model solves the following optimization problem:

$$\begin{aligned} \text{Max} \quad & u = u_m(\Delta\Pi_m)u_r(\Delta\Pi_r) = (\Delta\Pi_m)^{\lambda_m}(\Delta\Pi_r)^{\lambda_r} \\ \text{s.t.} \quad & \Delta\Pi_m \geq 0, \Delta\Pi_r \geq 0 \quad \text{and} \quad \Delta\Pi_m + \Delta\Pi_r = \Delta\Pi_{m+r}. \end{aligned}$$

The solution of this problem is $\Delta\Pi_m = \frac{\lambda_r}{\lambda_m + \lambda_r} \Delta\Pi_{m+r}$ and $\Delta\Pi_r = \frac{\lambda_m}{\lambda_m + \lambda_r} \Delta\Pi_{m+r}$, and this means the two parties will divide the joint extra-profit proportionally to their risk preference. When $\Delta\Pi_m$ and $\Delta\Pi_r$ have been determined, the manufacturer and the retailer can position themselves to some line $\bar{\Pi}_m = \Pi_m^* + \Delta\Pi_m$ (or equivalently, $\bar{\Pi}_r = \Pi_r^* + \Delta\Pi_r$) in Fig. 1. Specifically, this line can be expressed as

$$[4(k_r^2 + k_m^2)\beta w - k_r^2 t - k_m^2]/64\beta^2 = \Pi_m^* + \Delta\Pi_m,$$

or equivalently

$$t = 4(1 + k^2)\beta w - k^2 - 64(\Pi_m^* + \Delta\Pi_m)\beta^2/k_r^2.$$

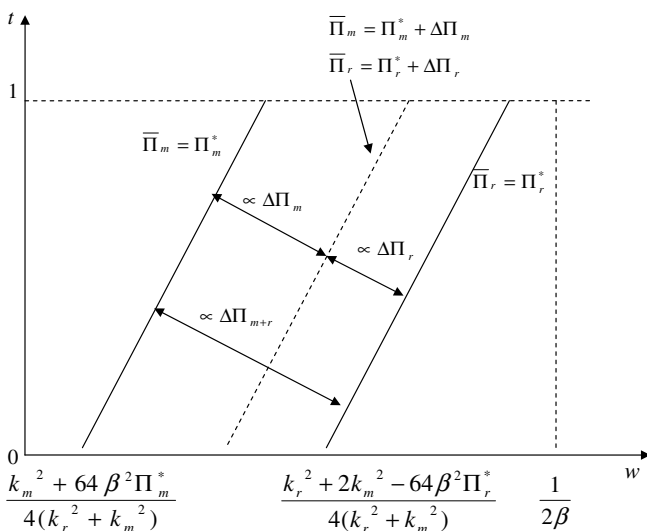
Thus, when both the manufacturer and the retailer have the same risk attitude ($\lambda_m = \lambda_r$), Nash’s model predicts that they will equally split the joint extra-profits, which is common knowledge in the bargaining literature. Otherwise, if one partner’s attitude is more risk-seeking than the other one’s, this partner will get more from the joint extra-profit, which is intuitively correct. We will not discuss in more details about this issue since it is out of the scope of the current paper. Similar remarks about determining exactly how the extra-profits should be divided can be found in Jewland and Shugan (1983, p.260).

6. Managerial implications and conclusions

This paper identifies the optimal equilibrium pricing and co-op advertising strategies in channel coordination between a manufacturer and a retailer. Using two game-theoretic models, we find that the cooperative model achieves better channel coordination and generates higher channel-wide profits than the non-cooperative, leader–follower model. In the cooperative model the advertising expenditures of both members are generally higher, but this does not come at the expense of consumers because the retail price is lower. The manufacturer and the retailer have to bargain over both the wholesale price and the manufacturer advertising participation rate to share the extra-profits achieved by cooperation. We identify the feasible solutions for this bargaining problem where the channel members can determine how to divide the extra-profits. Although the manufacturer’s price and advertising participation rate are not fully determined without specific parameter values, a linear equation links channel members at bargain equilibrium.

Our study makes the following contributions to the channel coordination literature:

- (i) We add to the scanty literature of game theoretical models that simultaneously optimize pricing and cooperative advertising decisions. We find close-form optimal solutions in both the Stackelberg model and the cooperative model where unique equilibriums exist.
- (ii) Most previous studies have used a single coordination instrument, such as wholesale price or two-part tariff. We find that when pricing and cooperative advertising are considered simultaneously, the coordination mechanism relies on both wholesale price and manufacturer’s participation rate, i.e., the pair of (w, t) values. Further, in Proposition 3 we establish the conditions and the feasible region where



(Here the symbol “∞” means “proportional to”)

Fig. 1. Feasible region for the bargaining problem.

the manufacturer and the retailer can bargain to divide the extra-profits accrued from coordination. This finding confirms the insightful observation made by Bergen and John (1997, p.362): “Coop allowances are not de facto wholesale price reductions. They are a distinct channel mechanism that has markedly different benefits from price in achieving coordination among retailers with intrabrand competition and local advertising spillovers”. Though we do not study intrabrand competition and advertising spillover in our simple supply chain, we show that both wholesale price and coop participation rate (w, t) are necessary to coordinate the channel. Based on his recent empirical study of 1446 cooperative advertising plans, Nagler (2006, p. 98) made a similar comment: “The results for manufacturer and retailer margins, taken together, suggest that cooperative advertising participation may serve as a mechanism for achieving improved channel coordination”.

Our model has some limitations. First, we assume a bilateral monopoly static model within which profits may be completely redistributed to either the manufacturer or the retailer. Adding competition, whether between manufacturers or between retailers, should enrich the model. Second, we use the linear demand-price function. As Choi (1991) has found, different demand-price functions may yield significantly different results and implications. Likewise, different functions of demand, price, and co-op advertising other than the multiplicative form in Eq. (1) may be used. Local advertising tends to emphasize price promotion, so there could be an overlapping or interactive effect between these two variables that calls for further research. Besides, we suggest the use of the traditional Nash approach to solve our bargain problem. Other interesting and less “traditional” models are available (Eliashberg, 1986; Kalai and Smorinsky, 1975) and might give some new solutions. Extending the channel structure to settings with multiple retailers and/or multiple manufacturers helps explore other dimensions of channel studies such as price competition, product differentiation, channel decentralization, among others. These issues have only been studied in the field of pricing (Choi, 1996; Ingene and Parry, 2000) but rarely in co-op advertising.

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Appendix

Proof of Proposition 1. Substituting Eqs. (9) and (10) into the expression of Π_m , the manufacturer's decision problem (11) becomes

$$\text{Max } \Pi_m = w \left(\frac{1 - \beta w}{2} \right) \left(\frac{k_r^2 (1 - \beta w)^2}{8\beta(1 - t)} + k_m \sqrt{A} \right) - \frac{tk_r^2 (1 - \beta w)^4}{64\beta^2 (1 - t)^2} - A$$

s.t. $0 \leq t \leq 1, 0 < w < 1/\beta$ and $A > 0$.

By taking $\partial \Pi_m / \partial A = 0, \partial \Pi_m / \partial t = 0$ and $\partial \Pi_m / \partial w = 0$, after algebraic simplification, we obtain

$$A = k_m^2 w^2 (1 - \beta w)^2 / 16, \tag{A1}$$

$$t = (5\beta w - 1) / (3\beta w + 1), \tag{A2}$$

$$8\beta(1 - 2\beta w)(1 - t)^2 k_m \sqrt{A} + (1 - 4\beta w)(1 - \beta w)^2 k_r^2 (1 - t) + (1 - \beta w)^3 k_r^2 t = 0. \tag{A3}$$

Substituting Eqs. (A1) and (A2) into (A3), after algebraic simplification, we have

$$(9k_r^2 + 16k_m^2)\beta^2 w^2 - 8k_m^2 \beta w - k_r^2 = 0. \tag{A4}$$

Since $0 < w < 1/\beta$, and making use of $k = k_m^2/k_r^2$, the sole solution w^* of (A4) is

$$w^* = \frac{4k + \sqrt{16k^2 + 16k + 9}}{\beta(9 + 16k)} = \frac{1/\beta}{\sqrt{16k^2 + 16k + 9} - 4k}. \tag{A5}$$

Recalling that t should be non-negative, thus Eq. (A2) holds only for $w \geq 1/5\beta$. For the case $w < 1/5\beta$, Eq. (A2) should be replaced by $t = 0$. Substituting Eq. (A1) and $t = 0$ into (A3), after algebraic simplification, we have

$$4(k - 1)\beta^2 w^2 + (5 - 2k)\beta w - 1 = 0. \tag{A6}$$

Since $k > 0$, we can easily prove (A6) has no solution under the condition $0 < w < 1/5\beta$. Therefore, (A5) together with (A1), (A2), (9) and (10) is the unique Stackelberg equilibrium for this game.

- (i) Noticing that $k > 0$, it can be easily seen that w^* expressed in (A5) is an increasing function of k and $\frac{\partial w^*}{\partial k} > 0$. Thus, we have $w^* > w^*|_{k=0} = 1/3\beta$ and $w^* < w^*|_{k \rightarrow \infty} = 1/2\beta$.
- (ii) Since t^* expressed in (A2) is an increasing function of w^* when $1/3\beta < w^* < 1/2\beta$, therefore it is also an increasing function of k , and thus we have $\frac{\partial t^*}{\partial k} > 0$ and $1/3 < t^* < 3/5$.
- (iii) According to (9), $p^* = (1 + \beta w^*)/2\beta$ is an increasing function of w^* , therefore it is also an increasing function of k , and thus we have $\frac{\partial p^*}{\partial k} > 0$ and $2/3\beta < p^* < 3/4\beta$.
- (iv) Similarly, since A^* expressed in (A1) is an increasing function of w^* when $1/3\beta < w^* < 1/2\beta$, and w^* is an increasing function of $k = k_m^2/k_r^2$, therefore A^* is an increasing function of k_m and a decreasing function of k_r , which also means $\frac{\partial A^*}{\partial k_m} > 0, \frac{\partial A^*}{\partial k_r} < 0$. Substituting (A2) into (10), we have $a^* = k_r^2 [(3\beta w^* + 1)(1 - \beta w^*)]^2 / 256\beta^2$ which is a decreasing function of w^* when $1/3\beta < w^* < 1/2\beta$. Thus, it is an increasing function of k_r and a decreasing function of k_m , which also means $\frac{\partial a^*}{\partial k_m} < 0, \frac{\partial a^*}{\partial k_r} > 0$.

This completes the proof of Proposition 1. \square

Proof of Proposition 2

- (i) From Proposition 1, we know that $2/3\beta < p^* < 3/4\beta$. Therefore, $\bar{p} = 1/2\beta < (3/4)p^* < p^*$.
- (ii) Within the range of $1/3\beta < w^* < 1/2\beta, A^* = k_m^2 [w^* (1 - \beta w^*)]^2 / 16$ is an increasing function of w^* , and $a^* = k_r^2 [(3\beta w^* + 1)(1 - \beta w^*)]^2 / 256\beta^2$ is a decreasing function of w^* . Thus, we have

$$A^* < k_m^2 [(1/2\beta)(1 - 1/2\beta)]^2 / 16 = k_m^2 / 256\beta^2 = (1/4)\bar{a} < \bar{a},$$

$$a^* < (3(1/3) + 1)^2 (1 - 1/3)^2 / 256\beta^2 = 1/144\beta^2 = (4/9)\bar{a} < \bar{a}.$$

- (iii) After substituting the expressions of p^*, a^*, A^* (as a function of w^*) into the expression of Π_{m+r} and algebraic simplification, the total profits of the whole channel under the Stackelberg game can be expressed as a function of w^* as follows:

$$\begin{aligned} \Pi_{m+r}^* &= p^*(1 - \beta p^*)(k_r \sqrt{a^*} + k_m \sqrt{A^*}) - a^* - A^* \\ &= (1 - \beta w^*)^2 [3k_r^2 \beta^2 (w^*)^2 + (10k_r^2 + 16k_m^2)\beta w^* + 3k_r^2] / 256\beta^2. \end{aligned} \tag{7}$$

Since $1/3\beta < w^* < 1/2\beta$, it is easily to see that Π_{m+r}^* is a decreasing function of w^* , and thus

$$\begin{aligned} \Pi_{m+r}^* &< \Pi_{m+r}^*|_{w^*=1/3\beta} = (20k_r^2/27 + 16k_m^2/27)/64\beta^2 \\ &< (20/27)(k_r^2 + k_m^2)/64\beta^2 = (20/27)\bar{\Pi}_{m+r} < \bar{\Pi}_{m+r}. \end{aligned}$$

This completes the proof of Proposition 2. \square

Proof of Proposition 3. After substituting the expressions of $(\bar{p}, \bar{a}, \bar{A})$ (Eqs. (13)–(15)) into the expression of Π_m and Π_r and algebraic simplification, the profits for the manufacturer and the retailer under cooperative relationship can be expressed as a function of (w, t) as follows, respectively:

$$\begin{aligned} \bar{\Pi}_m &= [4(k_r^2 + k_m^2)\beta w - k_r^2 t - k_m^2]/64\beta^2, \\ \bar{\Pi}_r &= [k_r^2 + k_r^2 t + 2k_m^2 - 4(k_r^2 + k_m^2)\beta w]/64\beta^2. \end{aligned}$$

Therefore, Eq. (17) is equivalent to

$$[4(k_r^2 + k_m^2)\beta w - k_r^2 t - k_m^2]/64 - \Pi_m^* \geq 0$$

and

$$[k_r^2 + k_r^2 t + 2k_m^2 - 4(k_r^2 + k_m^2)\beta w]/64\beta^2 - \Pi_r^* \geq 0,$$

which is also equivalent to $\frac{k_r^2 t + k_m^2 + 64\beta \Pi_m^*}{4(k_r^2 + k_m^2)} \leq w \leq \frac{k_r^2 t + k_r^2 + 2k_m^2 - 64\beta^2 \Pi_r^*}{4(k_r^2 + k_m^2)}$. It is also easy to check that the value of w in this range satisfies $0 < w < 1/2\beta = \bar{p}$ when $0 \leq t \leq 1$. This completes the proof of Proposition 3. \square

Derivations for some of the results in Table 1:

After substituting the expressions of p^*, a^*, A^*, t^* (as a function of w^*) into the expression of Π_m and Π_r and algebraic simplification, the profits for the manufacturer and the retailer under the Stackelberg game can be expressed as a function of w^* as follows, respectively:

$$\begin{aligned} \Pi_m^* &= (1 - \beta w^*)^2 [(9k_r^2 + 16k_m^2)(\beta w^*)^2 + 6k_r^2 \beta w^* + k_r^2]/256\beta^2, \\ \Pi_r^* &= (1 - \beta w^*)^2 [-(3k_r^2 + 8k_m^2)(\beta w^*)^2 + (2k_r^2 + 8k_m^2)\beta w^* + k_r^2]/128\beta^2. \end{aligned}$$

All the other results in Table 1 can easily be obtained (some of them are also shown in detail in the above proofs for the three propositions).

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