Cooperative Advertising with Bilateral Participation

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ABSTRACT

This paper studies cooperative (co-op) advertising strategies in a two-tier distribution channel and extends the popular unilateral participation strategy to bilateral participations. It is shown that a properly designed bilateral participation has several advantages over the unilateral participation. It is capable of coordinating the distribution channel under a very general demand function. In addition, when participation parameters are determined endogenously by channel members, the bilateral participation improves the channel efficiency and leads to a Pareto improvement over the corresponding unilateral participation.

Subject Areas: Cooperative Advertising, Bilateral Participation, Channel Coordination, Channel Efficiency, Marketing, and Dynamic Game.

INTRODUCTION

Consider a two-tier distribution channel with a manufacturer and a retailer, the modeling setting used by many researchers to study cooperative (co-op) advertising (Berger, 1972; Huang, Li, and

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Mahajan, 2002; Xie and Wei, 2009). The manufacturer conducts national advertisement to promote the brand name while the local retailer does local advertisement to boost short-term sales. It has been well established that if the manufacturer shares a portion of the retailer’s local advertising cost, the retailer will be motivated to spend more on its local advertisement and generate more sales. Such unilateral participation in co-op advertisement usually leads to a better channel performance, and eventually benefits the manufacturer, which justifies the manufacturer’s contribution to the retailer’s local advertisement. Following the same logic, the manufacturer’s national brand name promotion should also enhance the sales of retailer’s products associated with the brand name, which benefits the retailer. Should the retailer also contribute to the national advertisement that benefits itself? If so, we call such mutual contributions the bilateral participation in co-op advertising. To the best of our knowledge, this strategy has not been studied in the co-op advertising literature.

Clearly, bilateral participation is an extension of unilateral participation. As a result, a properly designed bilateral participation strategy should lead to a better channel performance. In this paper, we are interested in more precise comparisons between bilateral and unilateral participation strategies. In particular, we show that a properly designed bilateral participation strategy is capable of coordinating the distribution channel under a very general demand function. In addition, when participation parameters are determined endogenously by channel members, we show that the bilateral participation improves the channel efficiency and leads to a Pareto improvement over the corresponding unilateral participation under a popular demand function.

**BILATERAL PARTICIPATION AND CHANNEL COORDINATION**

Consider the two-tier distribution channel of a certain product with one manufacturer M and one retailer R. The short-term product sales $S(a_M, a_R)$ depends on both M’s national advertising
expenditure $a_M$ and R’s local advertising expenditure $a_R$. According to Huang, Li, and Mahajan (2002), “the local advertising and the national advertising perform different but complementary functions that have positive effects on the ultimate product sales”. We assume constant profit margins $\rho_M$ and $\rho_R$ for M and R respectively. Denote $r = \rho_M / \rho_R > 0$. Clearly, $r$ reflects the relative channel power between the two channel members.

For bilateral participation, M shares $t_M \in [0,1]$ portion of R’s local advertising cost $a_R$, and R shares $t_R \in [0,1]$ portion of M’s national advertising cost $a_M$. We call $t_M$ the manufacturer’s participation rate and $t_R$ the retailer’s participation rate. It is easy to verify that profits for M, R, and the whole distribution channel are

$$\Pi_M(a_M, a_R; t_M, t_R) = \rho_M S(a_M, a_R) - (1-t_R)a_M - t_M a_R, \quad (1)$$

$$\Pi_R(a_M, a_R; t_M, t_R) = \rho_R S(a_M, a_R) - t_R a_M - (1-t_M) a_R, \quad (2)$$

$$\Pi(a_M, a_R) = \Pi_M + \Pi_R = \rho S(a_M, a_R) - a_M - a_R, \quad (3)$$

respectively, where $\rho = \rho_M + \rho_R$ is the profit margin of the whole channel. Clearly, bilateral participation reduces to unilateral participation when $t_R = 0$.

Let the participation rates $t_M$ and $t_R$ be exogenously given and known to both channel members and they play the Stackelberg game led by M, i.e., for given $t_M$ and $t_R$, M first decides $a_M$, and R then chooses $a_R$. The following result shows that the bilateral participation strategy, with participation rates properly specified, can lead to fully channel coordination under a very general demand function. By channel coordination, we mean that the game’s equilibrium, i.e., the players’ advertising decisions, maximize the profit of the whole distribution channel.

**Proposition 1.** Suppose the manufacturer and the retailer play the Stackelberg game specified above and determine their advertising expenditures accordingly. If the demand function $S(a_M, a_R)$ is twice differentiable and strictly concave, then the distribution channel is coordinated if and
only if the channel members’ participation rates are given by
\[ t_M = r/(r+1) \quad \text{and} \quad t_R = 1/(r+1). \] (4)

**Proof. Necessity:** For an integrated distribution channel, equation (3) gives the overall channel profit. Since the market demand \( S(a_M, a_R) \) is strictly concave in \((a_M, a_R)\), there is a unique solution \((\bar{a}_M, \bar{a}_R)\) that solves the first order conditions \( \partial \Pi / \partial a_M = 0 \) and \( \partial \Pi / \partial a_R = 0 \), which are equivalent to
\[
\partial S( a_M, a_R ) / \partial a_M = \phi, \tag{5}
\]
\[
\partial S( a_M, a_R ) / \partial a_R = \phi. \tag{6}
\]
Therefore, the channel is coordinated if and only if players’ advertising expenditures in equilibrium coincide with \((\bar{a}_M, \bar{a}_R)\).

When M leads the Stackelberg game, R, at the second stage, will choose its best response advertising level \( a_R(a_M; t_M, t_R) \) according to M’s decision on \( a_M \), as well as the participation rates \((t_M, t_R)\). By solving the first order condition \( \partial \Pi_R / \partial a_R = 0 \), we have
\[
\rho_R \partial S( a_M, a_R ) / \partial a_R = 1. \tag{7}
\]
Comparing (6) and (7), we get
\[
t_M = 1 - \rho_R / \rho = \rho_M / \rho = r/(r+1), \tag{8}
\]
which proves the first half of equation (4).

Taking R’s optimal responses into consideration, M, at the first stage of the game, maximizes its profit \( \Pi_M(a_M, a_R; t_M, t_R) = \rho_M S(a_M, a_R(a_M; t_M, t_R)) - t_M a_R(a_M; t_M, t_R) - (1-t_R) a_M \). Solving the first order condition \( \partial \Pi_M / \partial a_M = 0 \), we obtain
\[
\rho_M \left[ \partial S(a_M, a_R) / \partial a_M + \partial S(a_M, a_R) / \partial a_R \frac{\partial a_R(a_M; t_M, t_R)}{\partial a_M} \right] = t_M \frac{\partial a_R(a_M; t_M, t_R)}{\partial a_M} + (1-t_R). \tag{9}
\]
Utilizing conditions (7) and (8), we can simplify equation (9) by
\[ \rho_M \hat{\gamma} S(a_M, a_R) / \hat{\alpha} a_M = 1 - t_R. \] (10)

Comparing (5) with (10), we get
\[ t_R = 1 - \rho_M / \rho = 1 / (r + 1), \] (11)
which proves the second half of equation (4).

**Sufficiency:** Under condition (4) channel members’ individual profits are induced consistent with (proportional to) the whole channel’s profit, as
\[ \Pi_M = \rho_M S(a_M, a_R) - (\rho_M \rho) a_M - (\rho_M \rho) a_R = (r/(r+1)) \Pi, \] (12)
\[ \Pi_R = \rho_R S(a_M, a_R) - (\rho_R \rho) a_M - (\rho_R \rho) a_R = (1/(r+1)) \Pi. \] (13)

In other words, condition (4) aligns player’s best interests with the whole system. Then, optimal individual actions will certainly maximize the total channel profit. This completes the proof.

To the best of our knowledge, Proposition 1 is among the first results that provide a clear incentive mechanism to coordinate the distribution channel in co-op advertising. It contrasts with conventional co-op advertising literature (e.g., Berger, 1972; Huang, Li, and Mahajan, 2002) that focuses on the Pareto improving bargaining process by sharing the additional profit when all channel members adopt the centralized decisions. In addition, the result remains to be true if R becomes the leader or if M and R play the Nash game instead. Based on Proposition 1, we have the following observations:

**Observation (i).** The unilateral participation strategy is in general not able to coordinate the distribution channel. This is because, according to (4), unilateral participation \((t_R = 0)\) coordinates the distribution channel if and only if \(\rho_R = 0\), i.e., when the retailer makes zero profit. As a result, unilateral participation strategy, such as that proposed in Huang, Li, and Mahajan (2002), cannot coordinate the distribution channel, since their demand function \(S(a_M, a_R) = a - \beta a_R^{-\tau} a_M^{-\delta}\), where \(\alpha, \beta, \gamma, \delta\) are all positive parameters, satisfies the assumptions of Proposition 1 and is a
special case of our general demand function.

Observation (ii). For the bilateral participation strategy to coordinate the distribution channel, it is necessary that participation rates \( t_M \) and \( t_R \) be exogenously specified. If channel members were to decide their participation rates endogenously, their decisions will deviate from those specified by (4). In fact, if \( R \) were allowed choose its participation rate \( t_R \), it would have chosen \( t_R = 0 \).

COMPARISON OF TWO PARTICIPATION STRATEGIES

In this section, we provide quantitative comparisons between two participation strategies under various game settings. In particular, we focus on situations where the participation rates are endogenously determined because in reality, the channel members, instead of a third party, are responsible to negotiate and come up with their own advertising decisions. We proceed by first specifying the demand function, and then defining different game settings.

For convenience of calculation, we choose the following additive demand function:

\[
S(a_M, a_R) = k_M \sqrt{a_M} + k_R \sqrt{a_R},
\]

where \( k_M, k_R \) are positive constants that reflect the efficacy of national and location advertisement, respectively. This same function was also used in Xie and Wei (2009). In particular, it is strictly increasing and concave with respect to \( a_M \) and \( a_R \) and models the commonly observed “advertising saturation effect” (Simon and Arndt, 1980). In addition, it also satisfies the assumption of Proposition 1. Denote \( k = k_M^2 / k_R^2 \) and it measures the relative effectiveness of national versus local advertising in generating customer demands. To further specify the game structures, we consider the following two issues:

Who Leads the Game?

Traditional literature in co-op advertising typically considers the manufacturer-led distribution channel, while studies regarding the retailer-led channel are sparse. Recently, more attentions are
paid to market structures where retailers retain equal or even more power than manufacturers (Buzzell, Ouelch, and Salmon, 1990; Huang, Li, and Mahajan, 2002; Buratto, Grosset, and Viscolani, 2007). Buratto, Grosset, and Viscolani (2007) studies two vertical co-op advertising programs by alternatively assigning the manufacturer and the retailer as the game leader to make final decisions on the participation rate. Their observation regarding channel member’s incentives to lead the distribution channel is partially verified by our model. However, there are significant differences; Buratto, Grosset, and Viscolani (2007) concludes that the retailer is the only member that always benefits by taking the lead; our study, on the other hand, reveals that the manufacturer also benefits by taking the lead. Moreover, we show that the distribution channel attains more profit only when it is led by the member with the higher profit margin.

Who Makes Which Decision?

The equilibrium of the game as well as the resulting profit for each player depends on the allocation of decision power to each player. In fact, inappropriate allocation of decision powers leads to trivial or unreasonable game results. For example, if the retailer is allowed to decide both the manufacturer’s participation rate \( t_M \) and its own advertising effort \( a_R \), the Stackelberg game will have a trivial and unreasonable outcome with \( t_M = 1 \) and \( a_R \to +\infty \). To exclude the trivial game settings, we propose two general decision rules:

Rule (i). The game follower should not decide either member’s participation rate.

Rule (ii). Any game player should make one of its own decisions: its own advertisement level or participation rate, but not both.

Notice that Rule (i) prevents the game follower from making decisions on channel members’ participation rates. This is because the follower tends to set the participation rates at the extreme levels. Given the game leader’s actions, the follower certainly prefers its own participation rate
to be zero and the other member’s participation rate to be one, which leads to trivial cases. Rule (ii), on the other hand, balances decision powers between channel members; each player should have sufficient but not excessive powers. Any violation of Rule (ii) will lead to a trivial game. For example, if a channel member decides both its participation rate and the other member’s advertising level, it will set its own participation rate at zero the other member’s participation rate at 100%. Similarly, if a channel member decides both its own advertising level and the other member’s participation rate, it will set its advertisement level at infinity and have the other member cover all the advertisement cost.

By screening all possible combinations for assigning decision variables (up to 40 cases in total) between channel members, we end up with four cases that satisfy both Rule (i) and Rule (ii). The associated game equilibrium and channel performance are summarized in Table 1. For simplicity, we denote each game setting by “Uni(Bi)/M(R)”, where Uni(Bi) means players adopt the unilateral(bilateral) participation strategy and M(R) indicates who leads the game.

For comparison purpose, we also presented two additional situations as benchmarks: the Full co-op case and the Null co-op case. The former represents the fully integrated distribution channel with all decisions centrally made; the latter describes the case where the channel members do not share each other’s advertisement costs. We will associate the over bar “~” with the Full co-op case.

To evaluate channel efficiencies, we introduce the profit ratio \( \eta = \frac{\Pi^*}{\Pi^*} \in [0, 1] \), in which the nominator is the channel profit under a specific game setting, and the denominator is the optimal profit for the integrated channel. We list all associated results in Table 2. For example, we have \( \frac{5}{9} \leq \eta \leq \frac{8}{9} \) at the second row of Table 2, indicating that the channel profit from the Null co-op can be as low as 56% of that of the coordinated channel ( \( \Pi^* \) ) and is limited to no more
than 89% of $\overline{\Pi}^*$. Here we assume that all efficiency calculations require the $r$ to stay in the range $\frac{1}{2} \leq r \leq 2$ so as to ensure all entries in Table 2 are valid.

Detailed derivations of equilibrium solutions and channel efficiency evaluations are quite similar for all cases. To save space, we only show the derivation for the Bi/M setting as an example. Under the Bi/M setting, R makes decisions on both the national and the local advertising expenditures ($a_M, a_R$); M controls the support rates for both types of advertising ($t_M, t_R$). For any given ($t_M, t_R$), R at the second stage of the game chooses its best responses $a_M(t_M, t_R)$ and $a_R(t_M, t_R)$ such that $\frac{\partial \Pi_r}{\partial a_M} = 0$ and $\frac{\partial \Pi_r}{\partial a_R} = 0$. As a result, we have $a_M(t_M, t_R) = \left(\frac{\rho \rho_4 \kappa^3}{2 \kappa} \right)^2$, and $a_R(t_M, t_R) = \left(\frac{\rho \rho_4 \kappa^3}{2 \kappa (1 - 1/\mu)} \right)^2$. Substituting the two into equation (1) and solving the first order conditions of $\Pi_M$ on $t_M$ and $t_R$ respectively, we obtain the optimal participation rates $t_M^* = \frac{2r - 1}{2r + 3}$ and $t_R^* = \frac{2}{2r + 3}$, where $r = \rho_M / \rho_R \geq \frac{1}{2}$. Then, players’ equilibrium advertising expenditures can be expressed as $a_M^* = a_M(t_M^*, t_R^*) = \left[\frac{\rho \rho_4 \kappa^3 (2r + 1)}{4r} \right]^2$, and $a_R^* = a_R(t_M^*, t_R^*) = \left[\frac{\rho \rho_4 \kappa^3 (2r + 1)}{4r} \right]^2$. Consequently, profit for M, R and the whole channel are computed as $\Pi_M^* = \frac{\rho \rho_4 \kappa^3}{4r} (k + 1) \left[r + \frac{1}{2}\right]^2$, $\Pi_R^* = \frac{\rho \rho_4 \kappa^3}{4r} (k + 1)(r + \frac{1}{2})$, and $\Pi^* = \frac{\rho \rho_4 \kappa^3}{4r} (k + 1) \left[1 + r - \frac{1}{2}\right]$. By definition, the channel efficiency under the Bi/M setting is given by $\eta = \Pi^*/\overline{\Pi}^* = 1 - \left[\frac{1}{2\kappa (r + 1)}\right]^2$, which increases in $r$. We can set $r = \frac{1}{2}$ (r=2) to compute the lower (upper) bound of $\frac{8}{9}$ ($\frac{33}{36}$).

By carefully comparing various game structures and their associated results in Table 1 and Table 2, we have following observations:

**Observation (iii).** Under both unilateral and bilateral participation strategies, although both members have the incentive to take the lead, the channel profit is higher when it is led by the member with the higher profit margin.
Observation (iii) can be verified by comparing the channel profits with \( r > 1 (\rho_M > \rho_R) \) and \( r < 1 (\rho_M < \rho_R) \) for cases when M takes the lead \((\text{Uni}/M, \text{Bi}/M)\) and when R takes the lead \((\text{Uni}/R, \text{Bi}/R)\) in Table 2.

**Observation (iv).** No matter who leads the channel, the bilateral strategy leads to higher channel profits than the corresponding unilateral one, even under the worst situation.

By comparing the optimal channel profits in Table 2 between the cases under unilateral participation and the corresponding cases under bilateral participation, it is straightforward to show that bilateral participation leads to higher channel profits. In addition, the system efficiency column in Table 2 shows clearly that the bilateral participation leads to higher system efficiency. Notice that while Observation (iv) is obvious when participation rates are specified exogenously, it is not as obvious when those decisions are made endogenously. It is also worthwhile noticing that according to Table 2 the distribution channel achieves almost perfect efficiency, between 89% and 97%, as long as the bilateral participation strategy is adopted.

Observation (iv) suggests that the bilateral participation leads to better channel performance. However, it is not clear if channel members have incentives to adopt this strategy. Our last result provides a positive answer to this concern: by switching to the bilateral participation both channel members will be better-off.

**Proposition 2.** For any given unilateral participation strategy with participation rate \( t_M \in [0,1] \) endogenously chosen, there exists a corresponding bilateral participation strategy \((\hat{t}_M, \hat{t}_R) \in [0,1]^2\) such that all members’ individual profits get better off, i.e., there always exists a strict Pareto improvement bilateral participation strategy.

**Proof.** For any unilateral participation rate \( t_M \in [0,1] \), channel members’ profit functions are, respectively, \( \Pi_M (t_M) = \frac{1}{4} k_R^2 \rho_R^2 \left[ k r^2 + \frac{2 t_M}{1 - t_M} - \frac{t_M}{(1 - t_M)^2} \right] \) and \( \Pi_R (t_M) = \frac{1}{4} k_R^2 \rho_R^2 \left[ 2 k r + \frac{1}{1 - t_M} \right] \). For any
bilateral participation strategy where M controls \((t_M, t_R) \in [0, 1]^2\) and delegates both advertising decisions to R, channel members’ profits are

\[
\Pi_M(\hat{t}_M, \hat{t}_R) = \frac{1}{4} k^2 \rho_R^2 \left[ \frac{2 r}{i_M} + \frac{2 r}{i_R} - \frac{1 - i_R}{i_R} - \frac{1 - i_M}{i_M} \right],
\]

and

\[
\Pi_R(\hat{t}_M, \hat{t}_R) = \frac{1}{4} k^2 \rho_R^2 \left[ \frac{1}{i_R} + \frac{1}{i_M} \right].
\]

For strict Pareto improvement, we are to find \( (\hat{t}_M, \hat{t}_R) \in [0, 1]^2 \) s.t.\(\Delta \Pi_M(t_M; \hat{t}_M, \hat{t}_R) = \Pi_M(\hat{t}_M, \hat{t}_R) - \Pi_M(t_M) > 0, \) and \(\Delta \Pi_R(t_M; \hat{t}_M, \hat{t}_R) = \Pi_R(\hat{t}_M, \hat{t}_R) - \Pi_R(t_M) > 0.\) Let \(\hat{t}_M = t_M,\) then we reduce \(\Delta \Pi_M = \frac{1}{4} k^2 \rho_R^2 \left[ r + \frac{1}{4} \right],\) which is always greater than zero, and \(\Delta \Pi_R(\hat{t}_R) = \frac{1}{4} k^2 \rho_R^2 \left[ \frac{1}{i_R} - (r - \frac{1}{i_R})^2 \right],\) which takes positive values if and only if \(t_{R1} < \hat{t}_R < t_{R2},\) where \(t_{R1} = \frac{1 + 2 r - \sqrt{1 + 4 r}}{2 r^2}\) and \(t_{R2} = \frac{1 + 2 r + \sqrt{1 + 4 r}}{2 r^2}\) are the two real roots of equation \(\Delta \Pi_R(\hat{t}_R) = 0.\)

Since \(0 < t_{R1} < 1 \leq t_{R2}\) holds for all \(r > 0,\) then we know that there always exists some \(\hat{t}_R \in [0, 1]\) that also belongs to the interval \((t_{R1}, t_{R2}).\) This completes the proof.

**CONCLUSIONS AND DISCUSSIONS**

While unilateral participation prevails in current co-op advertisement, this paper points out that it is in general not able to coordinate the distribution channel. In contrast, the extension to the bilateral participation will make it possible to coordinate the channel. In addition, bilateral participation will create a win-win situation for both the manufacturer and the retailer, making it a potentially promising and implementable strategy for co-op advertisement.

However, it remains to find a practical example of bilateral participation as described in this paper for reasons not addressed in our model: many retailers are not convinced about its benefit from the manufacturer’s national advertisement. Furthermore, in case of multiple retailers sharing the benefit of national advertisement, each individual retailer has no incentive to contribute due to the “free riding” effect. In order to make the bilateral participation strategy practical, an incentive mechanism needs to be designed for retailers to contribute for the benefit
they enjoy. As a future research, we are interested in designing and studying such incentive mechanism for a distribution channel with multiple retailers.

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**Bintong Chen** is a professor and vice dean of the Department of Business Administration, University of Delaware. ……
### TABLES

#### Table 1: Channel Member’s Performance under Different Game Settings.

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<th>Optimal participation rate(s) for R</th>
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<th>Equilibrium local Ad levels ((\cdot \frac{r^2k^2}{r}))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>((k+1)^2)</td>
<td>((r+1)^2)</td>
</tr>
<tr>
<td><strong>Null</strong></td>
<td>(M(R))</td>
<td>(a_M)</td>
<td>(a_R)</td>
<td>(\frac{2r-1}{2r+1})</td>
<td>(kr^2)</td>
<td>(1)</td>
</tr>
<tr>
<td><strong>Uni</strong></td>
<td>(M)</td>
<td>(a_M, t_M)</td>
<td>(a_R)</td>
<td>(\frac{2r}{r+2})</td>
<td>(kr^2)</td>
<td>((r+y)^2)</td>
</tr>
<tr>
<td><strong>Bi</strong></td>
<td>(R)</td>
<td>(a_M, a_R)</td>
<td>(t_M)</td>
<td>(\frac{2r}{r+2})</td>
<td>(kr^2)</td>
<td>((r+y)^2)</td>
</tr>
</tbody>
</table>

#### Table 2: Profit Functions and System Efficiency Estimation.

<table>
<thead>
<tr>
<th>Game setting</th>
<th>M’s optimal profit ((\cdot \frac{r^2k^2}{4}))</th>
<th>R’s optimal profit ((\cdot \frac{r^2k^2}{4}))</th>
<th>Optimal channel profit ((\cdot \frac{r^2k^2}{4}))</th>
<th>System efficiency</th>
<th>Conditions for the efficiency bounds</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-</td>
<td>((k+1)(r+1)^2)</td>
<td>1</td>
<td>(k=0, r=2) \text{ or } (k \to +\infty, r=\frac{1}{2})</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Null</strong></td>
<td>(kr^2 + 2r)</td>
<td>(2kr + 1)</td>
<td>(k(r^2 + 2r) + 2r + 1)</td>
<td>[(\frac{5}{9}, \frac{8}{9})]</td>
<td>(k=0, r=2) \text{ or } (k \to +\infty, r=\frac{1}{2})</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Uni/M</strong></td>
<td>(kr^2 + (r+y)^2)</td>
<td>(2kr + r + \frac{y}{2})</td>
<td>(k(r^2 + 2r) + (r+1)^2 - \frac{y}{4})</td>
<td>([\frac{5}{9}, \frac{35}{36}])</td>
<td>(k \to +\infty, r=\frac{1}{2})</td>
<td>(k=0, r=2)</td>
<td>-</td>
</tr>
<tr>
<td><strong>Uni/R</strong></td>
<td>(kr^2 + r(\frac{y}{2} + 1))</td>
<td>(2kr + (\frac{y}{2} + 1)^2)</td>
<td>(k(r^2 + 2r) + (r+1)^2 - \frac{y}{4})</td>
<td>([\frac{5}{9}, \frac{35}{36}])</td>
<td>(k \to +\infty, r=\frac{1}{2})</td>
<td>(k=0, r=\frac{1}{2})</td>
<td>-</td>
</tr>
<tr>
<td><strong>Bi/M</strong></td>
<td>((k+1)(r+y)^2)</td>
<td>((k+1)(r + \frac{y}{2}))</td>
<td>((k+1)((r+1)^2 - \frac{y}{4}))</td>
<td>([\frac{8}{9}, \frac{35}{36}])</td>
<td>(r=\frac{1}{2})</td>
<td>(r=2)</td>
<td>-</td>
</tr>
<tr>
<td><strong>Bi/R</strong></td>
<td>((k+1)(\frac{y}{2} + r))</td>
<td>((k+1)(\frac{y}{2} + 1)^2)</td>
<td>((k+1)((r+1)^2 - \frac{y}{4}))</td>
<td>([\frac{8}{9}, \frac{35}{36}])</td>
<td>(r=2)</td>
<td>(r=\frac{1}{2})</td>
<td>-</td>
</tr>
</tbody>
</table>