We consider a supply chain in which two suppliers compete for supply to a customer. Pricing and delivery-frequency decisions in the system are analyzed by two three-stage noncooperative games with different decision rights designated to the parties involved. The customer first sets the price (or delivery frequency) for each supplier. Then, the suppliers offer the delivery frequencies (or prices) simultaneously and independently. Finally, the customer determines how much demand to allocate to each of the suppliers. We show that delivery frequency, similar to delivery speed in time-based competition, can be a source of competitive advantage. It also allows firms that sell identical products to offer complementary services to the customer because she can lower her inventory with deliveries from more suppliers. In general, higher delivery frequencies lower the value of getting deliveries from the second supplier and therefore intensify price competition. Assuming the cost structures do not change and the suppliers are identical, we show that when the customer controls deliveries, she would strategically increase delivery frequencies to lower prices. The distortion in delivery frequencies is larger and the overall performance of the supply chain is lower when the customer, not the suppliers, controls deliveries. Moreover, the customer is better off under delivery competition, while the suppliers are better off under price competition.

(Pricing; Delivery; Logistics; Inventory; Just-in-Time; Supplier Management; Supply Chain Competition)

1. Introduction

Just-in-time has revolutionized manufacturing (Womack et al. 1990). Now logistics and distribution have become the critical links of supply chains. However, logistics and distribution management usually go beyond operational considerations because they involve different firms in a supply chain. For example, Toyota maintains a network of suppliers in close proximity to its manufacturing plants in North America and operates an elaborate logistics system (in conjunction with third-party logistics partners) to transport parts from the suppliers. The company manual of Toyota (1995) clearly states its objective of implementing just-in-time to logistics operations by “transporting parts in frequent shipments of small lots.” What are the advantages to a customer like Toyota if it controls delivery logistics in a supply relationship? Does just-in-time delivery (small-lot delivery) offer any benefits to the customer besides operational improvement in areas such as inventory and quality? How would the performance of individual firms as well as the system be affected if suppliers
are responsible for, and compete on, deliveries? We hope to shed light on these issues.

In this paper, we examine the role of delivery frequency in supplier competition and also examine how the nature of competition (price or delivery) and the decision rights (who is responsible for handling logistics and making price and/or delivery decisions) in a supply chain affect the performance of the system as well as that of individual firms. We posit a two-level supply chain model in which there is one customer with a constant demand of a product, and two suppliers. Production is instantaneous, an approximation of JIT manufacturing, but transportation is not. There is a cost associated with each batch of products delivered, regardless of its size. This fixed cost arising, for example, from the full-load problem of a vehicle, seems difficult to eliminate in the near future. Similar to the conventional EOQ model, these economies of scale in transportation breed a demand for inventory. However, in a supply chain setting it is the customer that will bear the cost of such inventory. Therefore, without the customer’s direct control of logistics or existence of competition in the upstream market, suppliers would have little incentive to deliver more frequently in smaller batches. We propose to study this issue in several models with different assumptions on how pricing and delivery-frequency decisions are made in the supply chain. Model 1 assumes that the suppliers handle delivery logistics and compete on delivery frequencies, while the customer determines prices and demand allocation. Model 2 assumes that the suppliers compete on prices, while the customer handles logistics and determines both delivery frequencies and demand allocation. We assume the firms that make delivery-frequency decisions are responsible for handling delivery logistics and bear corresponding costs, which is reasonable from a practical perspective. Model 3 assumes that the suppliers incur both the logistics and inventory costs and compete on price. Model 4 assumes that the supply chain is owned by one single firm that jointly determines delivery frequencies and demand allocation to minimize the system costs.

Our analysis highlights two important roles of delivery frequency in time-based competition. First, suppliers can actively seek a competitive advantage by delivering more often to their customers. When prices do not change, we show that increasing the delivery frequency of a supplier has two effects—a direct effect that increases the demand share of that supplier and a moderating effect on prices that makes the advantage of the low-price supplier more significant. Nevertheless, the demand share of a supplier always increases with his delivery frequency but decreases with that of his competitor. Second, delivery frequency allows suppliers to offer complementary services to the customer because she can reduce inventory with deliveries from more suppliers. Therefore, even when the suppliers are selling identical products, in equilibrium different suppliers may offer different delivery frequencies, charge different prices, and all have positive demand shares. One interesting insight from our second model is that when suppliers compete on prices, higher delivery frequencies may result in more intensive price competition, which is beneficial to the customer. Thus, even when the customer pays for the logistics costs, she has an incentive to strategically increase delivery frequencies so as to lower prices. From the supplier’s perspective, increasing the frequency of delivering to the customer has two effects. On the one hand, it makes this supplier more favorable because the associated inventory cost of the customer is reduced. On the other hand, it is undesirable to all the suppliers because price competition becomes more intensive. Our analysis shows that the profit of a supplier can indeed be lowered when the customer has more frequent shipments from him due to the adverse effect on prices. When the suppliers are identical, delivery frequencies are too high when the customer controls deliveries and too low when the suppliers compete on deliveries, as compared with the socially optimal case. The distortions in delivery frequencies are higher when they are determined by the customer instead of the suppliers, and therefore system performance deteriorates more in the first case. Moreover, the customer gains more power and therefore lowers her cost by controlling prices instead of deliveries. Thus, the customer is better off under delivery competition while the suppliers are better off under price competition.

Our work is related to the time-based competition literature, which examines delivery speed as a
source of competitive advantage. Stalk (1988) and Blackburn (1991) provide a comprehensive discussion of the managerial issues. Li (1992) investigates the role of inventory in time-based competition. Kalai et al. (1992), Loch (1994a), and Li and Lee (1994) study the effect of processing speed on price and firm’s performance, such as market share and profit. Loch (1994b) and Lederer and Li (1997) include scheduling as a strategic variable when customers are heterogeneous. More recently, So and Song (1998) examine competition with delivery-time guarantees. Hall and Porteus (2000) analyze a dynamic model that accounts for the customer’s switching behavior. Cachon and Harker (2002) consider competition between two firms with price- and time-sensitive demand, and investigate the impact of outsourcing on competition. The issue of price and capacity competition in congested markets has also been investigated in the economics literature. See, for example, Reitman (1991) and Deneckere and Peck (1995). All of this work considers the role of delivery speed in markets where customers incur delay costs. In this paper, we look at the role of delivery frequency (another time-based capability of a firm) in a supply chain setting where delay or inventory arises from economies of scale of transportation as opposed to stochastic variability, and where the customer incurs inventory holding cost. Another important difference of this paper with this literature is that the customer in our model has considerable power that influences the nature of competition.

Another related literature is that of order-quantity/lot-sizing coordination in supply chains when there are fixed ordering costs. Ordering decisions that are made by individual firms based on their self-interest are said to be coordinated when they optimize system performance. Most of the work in this literature focuses on deriving pricing schemes that can achieve coordination in a channel with one supplier and possibly multiple retailers. Crowther (1964), Monahan (1984), Lal and Staelin (1984), Lee and Rosenblatt (1986), and Dolan (1987) consider various quantity-discount schemes that can help coordinate ordering decisions. Weng (1995) and Chen et al. (2001) consider the case in which ordering and retail pricing decisions have to be jointly determined. Corbett and de Groote (2000) consider coordinating order quantities in a supplier-buyer relationship when the buyer’s cost is private information. This body of work emphasizes the issue of coordination, while horizontal competition has not been much explored. Our results complement those in this literature by demonstrating how the nature of horizontal competition and the decision rights in a supply chain affect the performance of the system as well as individual firms.

There is a growing literature on coordination and (vertical) competition along supply chains. See, for example, Lee et al. (1997), Cachon (1999), Lariviere (1999), and Tsay et al. (1999).

The remainder of this paper is organized as follows. Sections 2, 3, and 4 formulate and analyze Models 1, 2, and the socially optimal case (Models 3 and 4), respectively. In §5, we compare these models analytically for the case of identical suppliers. Finally, we provide our concluding remarks in §6. The proofs are given in the Appendix.

2. Model 1—Suppliers Handle Logistics and Compete on Delivery Frequencies

Consider a supply chain that consists of two suppliers and a customer. The two suppliers sell identical products (or parts) to the customer, who has a constant demand rate $D$. Our assumption of a constant demand rate is consistent with the practice of production smoothing by maintaining a constant master production schedule in just-in-time production. Supplier $i$ has a variable cost of production $c_i$, a unit price $p_i$, and an average delivery frequency $r_i$, where $i = 1, 2$. Although backlogging can be incorporated into our model, for simplicity we assume that no shortage is allowed or that shortage is prohibitively costly. We also assume that the delivery frequencies are averages; i.e., the suppliers do not necessarily deliver at equal time intervals. However, given the delivery frequencies $r_1$ and $r_2$, the customer staggered deliveries to minimize her inventory holding cost. Let $h$ be the unit holding-cost rate of the customer.

In Model 1, because the suppliers determine delivery frequencies, they are assumed to handle delivery logistics and incur the logistics-related costs. Let $k_1$
be the unit variable cost of transportation and \( K_i \) be the fixed cost per delivery for supplier \( i \). We assume that the fixed cost in production changeover incurred by a supplier is negligible compared with the fixed cost in transportation. This is reasonable because the changeover cost can be greatly reduced under just-in-time production, while the fixed cost in transportation is more difficult to change as it depends on the location of the supplier. This model is formulated as a three-stage noncooperative game. In the first stage, the customer announces the prices she is willing to pay to the suppliers. Given these prices, the suppliers then offer delivery frequencies simultaneously and independently in the second stage. Finally, given the prices and delivery frequencies, the customer determines demand allocation in the third stage. The three-stage game is solved backward by first analyzing the customer’s problem of demand allocation in the third stage, then the delivery competition subgame in the second stage, and finally the customer’s pricing problem in the first stage. Competition is examined based on the concept of Nash equilibrium.

Model 1 is applicable to the case in which the customer and the suppliers enter into long-term supply contracts and the customer has much higher bargaining power, so that she essentially dictates the prices. Because of logistics economics, it is more effective for the suppliers to handle their own deliveries. Moreover, the customer has the flexibility of changing demand allocation based on the suppliers’ short-term performance. One example is the dual-supplier policy employed by some Japanese manufacturers for parts that are highly dependent on the suppliers, and therefore a long-term supplier relationship is warranted.

2.1. Demand Allocation by the Customer in the Third Stage

Given prices \( p_1, p_2 \) and delivery frequencies \( r_1, r_2 \), the customer allocates demand and schedules deliveries to minimize the total purchasing and inventory holding costs. Let \( \alpha \) be the proportion of demand allocated to Supplier 1. Then the proportion of demand allocated to Supplier 2 is \( 1 - \alpha \). The delivery quantities \( Q_1 \) and \( Q_2 \) of the two suppliers are given by \( Q_1 = \alpha D/r_1 \) and \( Q_2 = (1 - \alpha)D/r_2 \). (For a given time period, if Supplier 1 delivers \( r_1 \) times for a total of \( \alpha D \) units, inventory due to Supplier 1 is minimized if his delivery quantities are the same; i.e., \( \alpha D/r_1 \). Similarly, inventory due to Supplier 2 is minimized if his delivery quantities are the same and given by \( (1 - \alpha)D/r_2 \).) Using the EOQ logic, obviously it is optimal for the next delivery to arrive when on-hand inventory just goes to zero. An example of the inventory build-up diagram with \( r_1 = 2/\text{month} \) and \( r_2 = 3/\text{month} \) is given in Figure 1.

In our model, the exact sequence of deliveries does not matter as long as the numbers of deliveries from the two suppliers are in the ratio of \( r_1 \) to \( r_2 \). For example, in Figure 1, the cost to the customer will remain the same if Supplier 1 delivers the first two batches (each of size \( Q_1 \)) while Supplier 2 delivers the next three batches (each of size \( Q_2 \)). As for the suppliers, we assume that their fixed costs in production changeover are negligible and they have ample

![Figure 1](image-url)
capacity, so that different delivery schedules will have little impact on their costs. In practice, it is conceivable that delivery schedules are jointly determined by the customer and the suppliers, probably through negotiations. Here, we just assume that they choose a schedule that is acceptable to the suppliers while satisfying the condition that the on-hand inventory of the customer goes to zero just before the next delivery arrives.

For a given time period (it is convenient to think in terms of a year and consider the annual costs) with \( r_1 \) and \( r_2 \) deliveries from the two suppliers, the total inventory due to Supplier 1 is \( r_1 \sigma_i^2/(2D) = \alpha^2D/(2r_1) \). Similarly, the total inventory due to Supplier 2 is \( (1-\alpha)^2D/(2r_2) \). Thus, the average total purchasing and inventory holding cost of the customer is given by

\[
C(\alpha) = p_1\alpha D + \frac{ha^2D}{2r_1} + p_2(1-\alpha)D + \frac{h(1-\alpha)^2D}{2r_2}.
\]

(1)

It is straightforward to show that the following allocation scheme \( \alpha^* \) minimizes \( C(\alpha) \):

\[
\alpha^* = \begin{cases} 
0 & \text{if } p_2 - p_1 \leq -\frac{h}{r_2}, \\
\frac{r_1}{r_1 + r_2} + \frac{r_1r_2}{r_1 + r_2} \frac{p_2 - p_1}{h} & \text{if } -\frac{h}{r_2} \leq p_2 - p_1 \leq \frac{h}{r_1}, \\
1 & p_2 - p_1 \geq \frac{h}{r_1}.
\end{cases}
\]

(2)

In the optimal allocation, Supplier 1 will have zero demand share if \( p_1 > p_2 + h/r_2 \), irrespective of his delivery frequency. This can be interpreted as a marginal cost condition. The total purchasing and inventory cost of buying the last unit from Supplier 2 is \( p_2 + h/r_2 \), where \( 1/r_2 \) is the amount of time this unit is held in inventory. If this is lower than the minimum cost from Supplier 1, \( p_1 \), then it is optimal to allocate all the demand to Supplier 2. When both suppliers have nonzero demand shares, the first term in \( \alpha^* \) accounts for the direct effect of delivery frequencies. A supplier with a higher delivery frequency has a larger competitive advantage. The second term accounts for the price effect moderated by delivery frequencies. Because \( r_1r_2/(r_1 + r_2) \) is increasing in \( r_1 \) and \( r_2 \), the price effect will be more significant when the delivery frequency of either supplier becomes higher. This is intuitive because the customer’s inventory holding cost associated with a supplier is convex decreasing in his delivery frequency. Suppose the delivery frequency of the high-price supplier increases. Both the positive direct effect and the negative price effect on the high-price supplier become more significant. Similarly, both the negative direct effect and the positive price effect on the low-price supplier become more significant. To see the overall effect, we can differentiate \( \alpha^* \) to get \( d\alpha^*/dr_1 = \frac{r_2}{r_1 + r_2^2} \) \( [1 + r_2(p_2 - p_1)/h] \) and \( d\alpha^*/dr_2 = \frac{r_1}{r_1 + r_2^2} \) \( [-1 + r_1(p_2 - p_1)/h] \). Thus, the demand share of a supplier is always increasing in his delivery frequency, but decreasing in that of his competitor.

2.2. Delivery Frequency Competition of the Suppliers in the Second Stage

Here, the suppliers are responsible for handling delivery logistics and compete on delivery frequencies in anticipation of the demand allocation in the third stage. Given prices \( p_1 \) and \( p_2 \), supplier \( i \) chooses delivery frequencies \( r_i \) to maximize his profit \( \Pi_i \). Without loss of generality, we only need to consider the case in which \( p_2 \geq p_1 \). From the result in §2.1, the profit functions of the two suppliers are given by

\[
\Pi_1(r_1, r_2) = \begin{cases} 
(p_1 - c_1 - k_1)\left(\frac{r_1}{r_1 + r_2} + \frac{r_1r_2}{r_1 + r_2} \frac{p_2 - p_1}{h}\right)D - r_1K_1 & \text{if } r_1 \leq \frac{h}{p_2 - p_1}, \\
(p_1 - c_1 - k_1)D - r_1K_1 & \text{otherwise};
\end{cases}
\]

(3)

\[
\Pi_2(r_1, r_2) = \begin{cases} 
(p_2 - c_2 - k_2)\left(\frac{r_2}{r_1 + r_2} - \frac{r_1r_2}{r_1 + r_2} \frac{p_2 - p_1}{h}\right)D - r_2K_2 & \text{if } r_1 \leq \frac{h}{p_2 - p_1}, \\
-r_2K_2 & \text{otherwise}.
\end{cases}
\]

(4)

We assume that \( p_1 \geq c_1 + k_1 \) and \( p_2 \geq c_2 + k_2 \). Otherwise, either both suppliers will not participate or only one supplier will participate in the second stage and the optimal delivery frequency chosen by the lone supplier will be zero. Both cases are undesirable to the
customer. Define the following best-response function for supplier $i$, where $i, j = 1, 2$ and $i \neq j$:

$$r_i^*(r_j) = \arg\max_{r_i \geq 0} \Pi_i(r_i, r_j).$$

Let $\Pi_1^i(r_2)$ and $\Pi_2^i(r_1)$ be the induced optimal profit functions; i.e., $\Pi_1^i(r_2) = \Pi_1(r_1^i(r_2), r_2)$ and $\Pi_2^i(r_1) = \Pi_2(r_1, r_2^i(r_1))$. To simplify the presentation of our results, define

$$\delta = \frac{p_2 - p_1}{h},$$

$$m_i = \frac{p_i - c_i - k_i}{K_i}, \quad i = 1, 2.$$

**Proposition 1.** The prices $p_1$ and $p_2$ are given.

(a) The unique Nash equilibrium of the delivery competition subgame is given by

$$r_1^* = \frac{1}{2\delta} \left( \frac{Dd_m m_2 - m_1 - m_2}{\sqrt{(Dd_m m_2 + m_1 - m_2)^2 + 4m_1 m_2}} + 1 \right),$$

$$r_2^* = \frac{1}{2\delta} \left( \frac{Dd_m m_2 + m_1 + m_2}{\sqrt{(Dd_m m_2 + m_1 - m_2)^2 + 4m_1 m_2}} - 1 \right),$$

$$\alpha^* = \frac{1}{2} + \frac{2}{2\sqrt{(Dd_m m_2 + m_1 - m_2)^2 + 4m_1 m_2}}.$$

(b) Sensitivity analysis:

(i) $r_1^*/r_2^*$ is increasing in $D$, $c_2$, $k_2$, and $K_2$ and decreasing in $h$, $c_1$, $k_1$, and $K_1$;

(ii) $\alpha^*$ is increasing in $D$ and decreasing in $h$, $c_1$, $k_1$, and $K_1$.

Part (a) shows that there is always a unique equilibrium. From the EOQ model, the delivery frequency in an integrated system is increasing in both the demand rate and the holding cost. Part (b) suggests that the delivery frequency of the low-price supplier is more sensitive to the demand rate, while that of the high-price supplier is more sensitive to the holding cost, so that the ratio of their delivery frequencies is increasing in the demand rate and decreasing in the holding cost. Due to these relative sensitivities of the delivery frequencies, the demand share of the low-price supplier is larger when the total demand rate is higher, but smaller when the holding cost is higher. The other results are intuitive.

### 2.3. Pricing Decisions of the Customer in the First Stage

Here, the customer chooses prices $p_1$ and $p_2$ to minimize her total purchasing and inventory holding costs in anticipation of the delivery-frequency selections and demand allocation in the later stages. As explained in the previous section, we only need to consider prices such that $p_1 \geq c_1 + k_1$ and $p_2 \geq c_2 + k_2$. Otherwise, the equilibrium delivery frequencies will be zero because at most one supplier participates in the second stage. By substituting the equilibrium delivery frequencies and demand allocation of the later stages into (1), the customer cost function can be shown to be given by

$$C(p_1, p_2) = p_1 D + \frac{hD}{2} \delta \left[ \frac{1}{2} - \frac{Dd_m m_2 + m_1 - m_2}{2\sqrt{(Dd_m m_2 + m_1 - m_2)^2 + 4m_1 m_2}} \right] + \frac{2m_2}{Dd_m m_2 + m_1 + m_2 - \sqrt{(Dd_m m_2 + m_1 - m_2)^2 + 4m_1 m_2}}.$$

The customer seeks to solve the following problem:

$$\min_{p_1 \geq c_1 + k_1, \quad p_2 \geq c_2 + k_2} C(p_1, p_2).$$

There is no explicit solution to the above problem. However, one can always search numerically for the optimal solution using a constrained optimization algorithm.

### 3. Model 2—Customer Handles Logistics and Suppliers Compete on Prices

The setup of Model 2 is similar to that of Model 1 except that now the customer is assumed to handle delivery logistics and determine delivery frequencies while the suppliers compete on prices. Let $k_C^i$ be the unit variable cost of transportation and $K_C^i$ be the fixed cost per delivery incurred by the customer when she buys from supplier $i$. This model is formulated as a three-stage noncooperative game where the customer announces delivery frequencies in the first stage, the suppliers then offer prices simultaneously and independently in the second stage, and the
customer determines demand allocation in the third stage.

Model 2 is applicable to the case in which the customer buys low-value, commodity-like parts from different suppliers, and it is more effective for the customer to coordinate and consolidate deliveries of these parts. One example is the supply network of Toyota described in §1. Setting up the supply network is a long-term decision by the customer. Suppliers on different routes of the network may compete for the supply of a standardized part. Because of the commodity nature of the part, the supply contracts are relatively short term and the suppliers compete mainly on prices.

3.1. Demand Allocation of the Customer in the Third Stage
Because the customer is assumed to handle delivery logistics, the average total purchasing, delivery, and inventory holding cost of the customer is given by

\[ C(\alpha) = (p_1 + k_1^c)\alpha D + \frac{h\alpha^2 D}{2r_1} + (p_2 + k_2^c)(1-\alpha)D + \frac{h(1-\alpha)^2 D}{2r_2} + r_1K_1^c + r_2K_2^c. \]  

(5)

It is straightforward to show that the following allocation scheme \( \alpha^* \) minimizes \( C(\alpha) \):

\[ \alpha^* = \begin{aligned} &0 &\text{if } p_2 + k_2^c - p_1 - k_1^c \leq -\frac{h}{r_2}, \\
&\frac{\frac{r_1}{r_1 + r_2} + \frac{r_2}{r_1 + r_2}}{\frac{h}{r_1}} &\text{if } -\frac{h}{r_2} \leq p_2 + k_2^c - p_1 - k_1^c \leq \frac{h}{r_1}, \\
&1 &\text{if } p_2 + k_2^c - p_1 - k_1^c \geq \frac{h}{r_1}. \end{aligned} \]  

(6)

Similar to Model 1, the optimal allocation depends on the relative delivery frequencies as well as the price differential moderated by the delivery frequencies, except that the customer now considers the total unit price of buying from a supplier; i.e. \( p_i + k_i^c \).

3.2. Price Competition of the Suppliers in the Second Stage
Given delivery frequencies \( r_1 \) and \( r_2 \) stipulated by the customer, supplier \( i \) chooses price \( p_i \) to maximize his profit \( \Pi_i \). Without loss of generality, we only need to consider the case in which \( c_2 + k_2^c \geq c_1 + k_1^c \). To avoid triviality, we only consider prices such that \( p_1 \geq c_1 \) and \( p_2 \geq c_2 \). From the optimal allocation in §3.1, the profit function of the supplier \( i \), where \( i, j = 1, 2 \) and \( i \neq j \), is given by

\[ \Pi_i(p_1, p_2) = \begin{cases} (p_i - c_i)D & \text{if } p_i + k_i^c - p_1 - k_1^c \geq \frac{h}{r_i}, \\
(p_i - c_i) \left( \frac{r_i}{r_1 + r_2} + \frac{r_2}{r_1 + r_2} \frac{p_i + k_i^c - p_1 - k_1^c}{h} \right) & \text{if } \frac{h}{r_j} \leq p_i + k_i^c - p_1 - k_1^c \leq \frac{h}{r_i}, \\
0 & \text{otherwise}. \end{cases} \]  

(7)

Define the following best-response function for supplier \( i \), where \( i, j = 1, 2 \) and \( i \neq j \):

\[ p_i^*(p_j) = \arg\max_{p_i \geq c_i} \Pi_i(p_1, p_2). \]

Let \( \Pi_1(p_2) \) and \( \Pi_2(p_1) \) be the induced optimal profit functions; i.e., \( \Pi_1(p_2) = \Pi_1(p_1^*(p_2), p_2) \) and \( \Pi_2(p_1) = \Pi_2(p_1, p_2^*(p_1)) \). Price competition in our model differs from the classical Bertrand competition in that the suppliers are differentiated due to their different delivery frequencies. The customer can always lower her inventory costs by aggregating the deliveries of both suppliers. Thus, it is possible for a supplier with both a higher variable cost and a lower delivery frequency to have a positive demand share. Because payoffs to the suppliers are continuous in their quoted prices, it is not surprising that there is a unique equilibrium in the price competition subgame. However, whether one or both suppliers can successfully sell to the customer depends on their costs. The equilibrium results are given in the following proposition.

**Proposition 2.** The delivery frequencies \( r_1 \) and \( r_2 \) are given.

(a) If \( c_2 + k_2^c - c_1 - k_1^c > 2h/r_1 + h/r_2 \), then the following is the unique Nash equilibrium of the price competition subgame:

\[ p_1^* = c_2 + k_2^c - k_1^c - \frac{h}{r_1}, \quad p_2^* = c_2, \]

\[ \alpha^* = 1, \]

\[ \Pi_1^* = \left( c_2 + k_2^c - c_1 - k_1^c - \frac{h}{r_1} \right)D, \quad \Pi_2^* = 0. \]
(b) If $0 \leq c_2 + k_2^c - c_1 - k_1^c \leq 2h/r_1 + h/r_2$, then the following is the unique Nash equilibrium of the price competition subgame:

$$p_i^* = \frac{1}{3} \left( 2c_i + \frac{h}{r_i} + c_j + \frac{2h}{r_j} + k_j^c - k_i^c \right),$$

$$\alpha^* = \frac{1}{3h} r_1 r_2 \left( c_2 + k_2^c + \frac{h}{r_1} + \frac{2h}{r_2} - c_1 - k_1^c \right),$$

$$\Pi_i^* = \frac{D}{9h} r_1 r_2 \left( c_j + k_j^c + \frac{h}{r_j} + \frac{2h}{r_j} - c_i - k_i^c \right)^2,$$

where $i, j = 1, 2$ and $i \neq j$.

Part (a) shows that if the variable cost differential is higher than a critical value, the high-cost supplier will have zero demand share. It may seem counterintuitive that this critical value is decreasing in the delivery frequencies of both suppliers as it implies that the high-cost supplier is more likely to have zero demand share when his delivery frequency becomes higher. This can be explained by the optimal allocation in the third stage, which shows that price competition becomes more intensive and the cost advantage of the low-cost supplier becomes more significant when the delivery frequencies become higher. Here, the variable cost of the supplier is defined from the customer’s perspective and is the sum of the supplier’s variable production cost and the customer’s variable logistics cost of delivering from that supplier. A supplier may have a competitive disadvantage due to either his high production cost or, say, his remote location from the customer, which results in prohibitively expensive delivery costs. When the customer buys solely from the low-cost supplier, the marginal cost of the last unit is $p_i + k_i^c + h/r_i$. Alternatively, buying this last unit from the high-cost supplier incurs a minimum cost of $c_2 + k_2^c$ because there is no inventory holding. The low-cost supplier optimally sets his price $p_i^*$ so that the high-cost supplier cannot lower his price further to be competitive. The condition $c_2 + k_2^c - c_1 - k_1^c > 2h/r_1 + h/r_2$ ensures that the low-cost supplier maximizes his profit by getting all the demand. This condition also implies that costs are the primary drivers that determine the outcome of price competition, while delivery frequencies play only a secondary role. Part (b) says that when both suppliers contract with the customer, they will generally charge different prices depending on their costs and their delivery frequencies.

**Proposition 3.** The delivery frequencies $r_1$ and $r_2$ are given.

(a) If $c_2 + k_2^c - c_1 - k_1^c \geq 2h/r_1 + h/r_2$, then

(i) $p_i^*$ is increasing in $c_i$, $c_j$, $k_j^c$, and $h$ and decreasing in $k_i^c$ and $r_i$;

(ii) $\alpha^*$ is increasing in $c_2$, $k_2^c$, and $r_1$ and decreasing in $c_1$, $k_1^c$, and $h$.

(iii) $\Pi_i^*$ is increasing in $c_2$, $k_2^c$ and $D$ and decreasing in $c_1$, $k_1^c$, and $h$.

(b) If $0 \leq c_2 + k_2^c - c_1 - k_1^c \leq 2h/r_1 + h/r_2$, then

(i) $p_i^*$ is increasing in $c_i$, $c_j$, $k_j^c$, and $h$ and decreasing in $k_i^c$, $r_i$, and $r_j$;

(ii) $\alpha^*$ is increasing in $c_2$, $k_2^c$, and $r_1$ and decreasing in $c_1$, $k_1^c$, and $h$;

(iii) $\Pi_i^*$ is increasing in $c_2$, $k_2^c$ and $D$ and decreasing in $c_1$, $k_1^c$, and $r_2$.

For the single-supplier equilibrium in part (a), both the price and the profit of the low-cost supplier are decreasing in the holding cost and increasing in the delivery frequency. As discussed earlier, the marginal cost of buying from the low-cost supplier, $p_i + k_i^c + h/r_i$, cannot exceed the cost of the high-cost supplier. Therefore, a higher inventory cost due to either a higher holding cost or a lower delivery frequency (and a higher variable logistics cost $k_i^c$) results in a lower price and, hence, a lower profit for the low-cost supplier.

For the dual-supplier equilibrium in part (b), it would be interesting to examine the effect of several parameters. A higher holding cost lessens the intensity of price competition so that the prices of both suppliers increase. However, this makes the cost advantage of the low-cost supplier less significant, and therefore his demand share decreases. As
a result, the profit of the low-cost supplier is not
monotone in the holding cost, while that of the high-
cost supplier always increases with the holding cost.
Suppose the delivery frequency of the low-cost sup-
plier increases. This increases his delivery advantage
as well as his cost advantage due to more intensive
price competition, resulting in a higher demand
share. However, prices decrease due to more intensive
price competition. Because of these opposing effects
on profit, the profit of the low-cost supplier is not
monotone in his delivery frequency. The profit of
the high-cost supplier always decreases because both
his price and his demand share decrease when the
delivery frequency of the low-cost supplier increases.
Now suppose the delivery frequency of the high-
cost supplier increases. This increases his delivery
advantage, but also increases the cost advantage
of the low-cost supplier. Thus, the demand shares
of the suppliers are not monotone in the delivery
frequency of the high-cost supplier. When the delivery
frequency of the low-cost supplier is sufficiently high,
his demand share actually increases with the delivery
frequency of the high-cost supplier, showing that
the cost effect now dominates the effect of delivery
frequency. This is consistent with part (a) of Pro-
position 2, which says that for the low-cost supplier to
get all the demand, it is more likely to occur with a
higher delivery frequency of the high-cost supplier.
Again, prices always decrease when delivery frequen-
cies increase due to more intensive price competition.
The profits of both suppliers are decreasing in the
delivery frequency of the high-cost supplier, though
its effect on their demand shares is not uniform. In
this case, the effect of price reductions due to a higher
delivery frequency of the high-cost supplier is more
significant than that of changes in demand shares. The
other results are intuitive.

3.3. Delivery-Frequency Decisions of the
Customer in the First Stage
Again, without loss of generality, assume $c_2 + k_2^C \geq c_1 + k_1^C$. By substituting the equilibrium prices and
demand allocation of the price competition subgame
into (5), the customer cost function can be shown to be

$$
C(r_1, r_2) = \begin{cases} 
\left( c_2 + k_2^C \right) D + \frac{hD}{9} \left[ \frac{h}{2} \left( c_1 + k_1^C \right) r_1 + \left(c_2 + k_2^C \right) r_2 \right] \\
\left( c_2 + k_2^C \right) D + \frac{hD}{9} \left[ \frac{h}{2} \left( c_1 + k_1^C \right) r_1 + \left(c_2 + k_2^C \right) r_2 \right] \\
+ \frac{11}{2} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) + \frac{1}{2} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \\
- \frac{\left(c_2 + k_2^C - c_1 - k_1^C \right)^2 r_1 r_2}{h^2 (r_1 + r_2)} \\
+ r_1 K_1^C + r_2 K_2^C & \text{otherwise.}
\end{cases}
$$

The customer seeks to solve the following problem:

$$
\min_{r_1 \geq 0, r_2 \geq 0} C(r_1, r_2).
$$

The optimal delivery frequencies can be obtained by
first finding the optimal solutions in the two regions
defined by $c_2 + k_2^C - c_1 - k_1^C \geq 2h/r_1 + h/r_2$ and $0 \leq c_2 +
\frac{1}{2} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) + \frac{1}{2} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \\
- \frac{\left(c_2 + k_2^C - c_1 - k_1^C \right)^2 r_1 r_2}{h^2 (r_1 + r_2)} \\
+ r_1 K_1^C + r_2 K_2^C & \text{otherwise.}
\end{cases}
$$

The optimal delivery frequencies can be obtained by
first finding the optimal solutions in the two regions
defined by $c_2 + k_2^C - c_1 - k_1^C \geq 2h/r_1 + h/r_2$ and $0 \leq c_2 +
\frac{1}{2} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) + \frac{1}{2} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \\
- \frac{\left(c_2 + k_2^C - c_1 - k_1^C \right)^2 r_1 r_2}{h^2 (r_1 + r_2)} \\
+ r_1 K_1^C + r_2 K_2^C & \text{otherwise.}
\end{cases}
$$

In the second region, it can be shown that $C(r_1, r_2)$ is
jointly convex in $r_1$ and $r_2$, and therefore the optimal
solution can be solved from the first-order conditions.
This result is stated, without proof, in the following
proposition.

**Proposition 4.** For $0 \leq c_2 + k_2^C - c_1 - k_1^C \leq 2h/r_1 + h/r_2$, $C(r_1, r_2)$ is jointly convex in $r_1$ and $r_2$, and the optimal
solution can be obtained by solving the following first-order conditions:

\[
\begin{align*}
\frac{11}{r_1} & + \frac{2(c_2 + k_2^C - c_1 - k_1^C) r_2}{h(r_1 + r_2)^2} \\
& + \frac{2(c_2 + k_2^C - c_1 - k_1^C) r_2}{h^2 (r_1 + r_2)^2} = 18k_1^C \frac{hD}{hD},
\end{align*}
\]
4. The Socially Optimal Case

In this section, we consider two models that lead to the socially optimal solution. For Model 3, we consider the case in which the suppliers hold inventory at a warehouse adjacent to the customer and deliver to the customer in a just-in-time fashion. The suppliers may deliver infrequently to the warehouse due to long traveling distances. This is similar to the practice of supplier hub, where a third-party logistics provider manages inventory for the suppliers and the suppliers are responsible for the logistics and inventory costs. When the suppliers compete mainly on prices, this is the classical Bertrand competition. The supplier who gets all the demand will choose his frequency of delivering to the warehouse based on the EOQ solution. Without loss of generality, assume \((c_1 + k_1)D > \sqrt{2K_1 Dh} < (c_2 + k_2)D + \sqrt{2K_2 Dh}\). The unique equilibrium is for Supplier 1 to charge \(p_1\) such that \(p_1 D = (c_2 + k_2)D + \sqrt{2K_2 Dh}\), get all the demand from the customer, and obtain the profit \((c_2 + k_2)D + \sqrt{2K_2 Dh} - (c_1 + k_1)D - \sqrt{2K_1 Dh}\). The cost of the customer is given by \((c_2 + k_2)D + \sqrt{2K_2 Dh}\). Because the suppliers fully internalize the inventory costs, there is no distortion in delivery frequency and the socially optimal solution can be achieved. When the suppliers are identical, the customer is always better off compared with either delivery or price competition because it is easy to see that she can extract all the profit from the supply chain. However, our framework here is mainly concerned with the incentives relating to competition and the designation of decision rights, but is not rich enough to account for the operational improvements due to supplier consolidation. A more detailed model is necessary to fully investigate the issue of supplier competition for the case of supplier hub.

For Model 4, we assume that the customer and the suppliers belong to the same firm, which seeks to minimize the total production, delivery, and inventory cost in the supply chain. This is the case of an integrated firm. The integrated firm has to solve the following problem:

\[
\min_{r_1 \geq 0, r_2 \geq 0, \text{or} \leq 1} C(a, r_1, r_2),
\]

where

\[
C(a, r_1, r_2) = (c_1 + k_1)D + \frac{ra^2 D}{2r_1} + r_1 K_1 + (c_2 + k_2)(1-a)D + \frac{h(1-a)^2 D}{2r_2} + r_2 K_2
\]

\[
= a \left[ (c_1 + k_1)D + \frac{hQ_1}{2} + \frac{K_1 D}{Q_1} \right] + (1-a) \left[ (c_2 + k_2)D + \frac{hQ_2}{2} + \frac{K_2 D}{Q_2} \right],
\]

and \(Q_1 = aD/r_1, Q_2 = (1-a)D/r_2\) are the delivery quantities. Given \(a\), the optimal delivery quantity of each supplier is given by the classical EOQ solution that minimizes the total inventory holding and fixed delivery cost. The customer cost function becomes

\[
C(a) = a \left[ (c_1 + k_1)D + \sqrt{2K_1 Dh} \right] + (1-a) \left[ (c_2 + k_2)D + \sqrt{2K_2 Dh} \right].
\]

Obviously, it is optimal to allocate all demand to the supplier with the lower total cost, and hence the minimized system cost is given by

\[
\min \left[ (c_1 + k_1)D + \sqrt{2K_1 Dh}, (c_2 + k_2)D + \sqrt{2K_2 Dh} \right].
\]

5. Comparing the Models with Identical Suppliers

In this section, we compare the four models to shed light on how the nature of competition and the designation of decision rights affect the performance of individual firms as well as the supply chain. To focus on the effect of the above-mentioned two factors and to make the models comparable, we assume that the cost structures of the models are the same. For example, \(k_1 = k^C_1\) and \(K_1 = K^C_1\) due to, say, the use of the same transportation technologies. We also assume that the suppliers are identical; i.e., \(c_1 = c_2 = c, k_1 = k_2 = k_1^C = k_2^C = k\), and \(K_1 = K_2 = K_1^C = K_2^C = K\). In this
case, Models 3 and 4 are identical. Based on the analysis of §§2, 3, and 4, the results for the case of identical suppliers can be derived and they are summarized in Table 1. To derive the optimal prices of Model 1, we assume that the customer charges the same prices for both suppliers. We computed numerically the optimal prices of Model 1 without such an assumption for a large number of problems and found that the optimal prices are always the same. For Model 4 (integrated system), any demand allocation is optimal, the delivery quantities of both suppliers are given by the EOQ solution $\sqrt{2KD}/h$, and the resulting total delivery frequency is $\sqrt{Dh/2K}$. We choose an (0.5, 0.5) allocation for the ease of comparison with the other two models.

First, consider system performance. Delivery competition with prices determined by the customer leads to the highest inventory cost but the lowest logistics cost. Price competition with deliveries controlled by the customer leads to the lowest inventory cost, but the suppliers incur logistics cost while inventory cost only indirectly affects their profits through demand allocation, they choose delivery frequencies that are lower than optimal for the system. Under price competition, although the customer incurs the logistics cost, she has an incentive to deliver more often from both suppliers so as to induce more intensive price competition. Thus, the resulting delivery frequencies are higher than optimal. Under delivery competition, delivery frequency is 29% below the optimal value, while under price competition, it is 124% above. The larger distortion in delivery frequency explains why the total system cost is the highest under price competition.

Now consider individual performance. The customer is better off (lower cost) under delivery competition, while the suppliers are better off (higher profit) under price competition. This can be explained by the relative power of the customer in these two models. In Model 1, the customer determines prices by anticipating the outcome of the delivery competition in the later stage. In Model 2, the customer determines delivery frequencies by anticipating the outcome of the price competition in the later stage. In this case, she uses delivery frequencies to indirectly influence prices. Obviously, it is more effective for the customer to use pricing instead of delivery frequency to lower her costs. Indeed, under price competition each supplier earns a profit that is more than 150% higher than that under delivery competition.

When the suppliers are not identical, we are not able to analytically solve for the first-stage optimal solutions of Models 1 and 2. We have performed a

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comprehensive numerical study for the case of non-
identical suppliers. Our results show that most of the
insights from the case of identical suppliers remain
valid, except when the cost advantage (in either fixed
cost or variable cost) of the low-cost supplier is very
large. In this case, under price competition it is pos-
sible for the delivery frequency to be lower than the
socially optimal case. Moreover, under price compe-
tition the system cost may be lower, while both the
low-cost supplier and the customer may be better off
compared with delivery competition. Details of the
numerical results can be found in Ha et al. (2002).

6. Conclusion
We have investigated the role of delivery frequency
on supplier competition. In general, higher delivery
frequency is a source of competitive advantage (direct
effect), and it may also lead to more intensive price
competition (moderating effect). The delivery and
price competition subgames of Models 1 and 2 are
of independent interest when either prices or deliv-
ery frequencies are exogenously given. For exam-
ple, two distributors may sell substitutable products
to a retailer. Both the distributors and the retailer
have little power to control the wholesale prices,
which are largely determined by the manufacturers
on a regional basis. In this case, the distributors will
compete on delivery frequencies as in the delivery
competition subgame of Model 1. Now consider a
manufacturer who operates a parts delivery network
that involves a large number of suppliers. The deliv-
ery frequencies of different routes in the network are
relatively rigid due to long-term contractual arrange-
ments between the manufacturer and the third-party
logistics companies. When suppliers are selected for
a new part, they may compete according to the price
competition subgame of Model 2.

We have also shown how the nature of competi-
tion and the designation of decision rights in a supply
chain affect the performance of the system as well as
individual firms. Our results are obtained by assum-
ing that the cost structures of the models are the same.
In practice, one has to interpret these results by taking
the validity of this assumption into consideration. For
example, one insight from §5 is that when the cus-
tomer controls deliveries while the suppliers compete
on prices, price competition leads to higher distor-
tions in delivery frequencies and the total cost of the
supply chain is higher. However, if there are signifi-
cant economies of scale for the customer to coordinate
deliveries (due to the existence of suppliers of other
parts) so that the logistics costs are much lower under
price competition, the total supply chain cost may
be lower in spite of the larger distortions in delivery
frequency. In this case, while the qualitative insight
of our analysis remains valid, one has to take other
factors (such as changes in the cost structures) into
consideration.

While Model 3 can achieve the socially optimal
solution, sometimes Models 1 and 2 may result in
a lower overall supply chain cost due to different
cost structures. For example, there may be significant
economies of scale when the customer is responsible
for deliveries or holding inventory. Moreover, the sup-
pliers need incentives to bear the additional cost of
holding inventory and providing just-in-time deliver-
ies to the customer. The issue of designing the optimal
supply chain structure under supplier competition is
interesting and will be left for further research.

We have considered competition on only one of the
two dimensions (price or delivery frequency). Some-
times suppliers may compete on price and delivery
frequency simultaneously. However, this is beyond
the scope of this paper, and we will leave this for
future research. We have focused on the issues of
pricing and delivery frequency in supply chains with
horizontal competition. It would be interesting to
extend our framework to address other issues such
as scheduling of order deliveries, demand uncertain-
ty, asymmetric information, and process improve-
ments. The role of third-party logistics suppliers in a
traditional supplier-customer relationship is another
related issue that is worthy of further research.

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Appendix

Proof of Proposition 1. We first prove the following lemma, which characterizes some useful properties of the best-response functions.

**Lemma 1.** (a) For Supplier 1, if \( \delta m_1 D < 1 \), then
\[
 r^*_1(r_2) = \begin{cases} \sqrt{m_1 D} \sqrt{r_2 (1 + \delta r_2)} - r_2 & \text{if } 0 \leq r_2 \leq \frac{m_1 D}{1 - \delta m_1 D}, \\ 0 & \text{otherwise}, \end{cases}
\]
\[
 \Pi^*_1(r_2) = \begin{cases} K_1 \left( \sqrt{m_1 D} \sqrt{1 + \delta r_2} - \sqrt{r_2} \right)^2 & \text{if } 0 \leq r_2 \leq \frac{m_1 D}{1 - \delta m_1 D}, \\ 0 & \text{otherwise}. \end{cases}
\]

\( r^*_1(r_2) \) is concave increasing in \( r_2 \) for
\[
 0 \leq r_2 \leq \frac{1 - \sqrt{1 - \delta m_1 D}}{2 \sqrt{1 - \delta m_1 D}}.
\]

Proof of Lemma 1. For part (a), first consider the case \( \delta m_1 D < 1 \). From (3), \( \Pi_1(1/\delta, r_2) = K_1 (\delta m_1 D - 1)/\delta \) and \( \Pi_1(r_1, r_2) \) is concave in \( r_1 \) for \( r_1 \leq 1/\delta \). When \( r_1 \geq m_1 D/(1 - \delta m_1 D) \), by differentiating \( \Pi_1(r_1, r_2) \) with respect to \( r_1 \), it is easy to show that \( \Pi_1(r_1, r_2) \) is maximized at \( \sqrt{m_1 D} \sqrt{r_2 (1 + \delta r_2)} - r_2 \), which is between 0 and 1/\( \delta \), and the induced optimal profit is
\[
 K_1 \left( \sqrt{m_1 D} \sqrt{1 + \delta r_2} - \sqrt{r_2} \right)^2 - \left( \sqrt{1 - \delta m_1 D} \right)^2 > 0.
\]

When \( r_1 > m_1 D/(1 - \delta m_1 D) \), \( \Pi_1(r_1, r_2) \) is decreasing in \( r_1 \). Therefore, \( \Pi_1(r_1, r_2) \) is maximized at \( r_1 = 0 \) and the induced optimal profit is zero. Now consider the case \( \delta m_1 D \geq 1 \). Similarly, we can show that \( \Pi_1(r_1, r_2) \) is maximized at \( \sqrt{m_1 D} \sqrt{r_2 (1 + \delta r_2)} - r_2 \) when \( r_2 \leq 1/\delta \). Otherwise, \( \Pi_1(r_1, r_2) \) is increasing in \( r_1 \) up to 1/\( \delta \) and then decreasing. Therefore, it is maximized at \( r_1 = 1/\delta \). The other results can be obtained by inspecting the first and second derivatives of \( r^*_1(r_2) \) and \( \Pi^*_1(r_2) \) with respect to \( r_2 \). The proof of part (b) is similar to that of part (a).

Details omitted. \( \square \)

The best-response functions \( r^*_1(r_2) \) and \( r^*_2(r_1) \) are plotted in Figure 2.

Now we are ready to prove Proposition 1. For part (a), we need to show that there is a unique solution to the following equations:
\[
 r_1 = \sqrt{m_1 D} \sqrt{r_2 (1 + \delta r_2)} - r_2, \quad (A.1)
\]
\[
 r_2 = \sqrt{m_2 D} \sqrt{r_1 (1 - \delta r_1)} - r_1. \quad (A.2)
\]

Let
\[
 x = \frac{1/r_1 - \delta}{1/r_2 + \delta}, \quad y = \frac{\delta}{1/r_1 + \delta}. \quad (A.3)
\]

By substituting (A.3) into (A.1) and (A.2) and rearranging terms, we get
\[
 \frac{\delta m_1 D}{y} (x + y)^2 = (x + 1)^2, \quad (A.4)
\]
\[
 \frac{\delta m_2 D}{y} (1/y)^2 = (x + 1)^2. \quad (A.5)
\]

From (A.4) and (A.5), we have
\[
 y = \frac{\sqrt{m_1 D} \sqrt{m_1 D} - x}{\sqrt{m_2 D} \sqrt{m_2 D} + x} \quad (A.6)
\]

We do not consider the other root
\[
 y = \frac{\sqrt{m_1 D} \sqrt{m_2 D} - \sqrt{m_1 D} \sqrt{m_2 D}}{\sqrt{m_1 D} \sqrt{m_2 D} + \sqrt{m_1 D} \sqrt{m_2 D}} \quad (A.7)
\]

because \( y \leq 1 \). Now, substituting (A.6) into (A.5) and simplifying terms, we get
\[
 x + \left( \frac{\sqrt{m_1 D} \sqrt{m_1 D} + \sqrt{m_2 D} \sqrt{m_2 D}}{\sqrt{m_1 D} \sqrt{m_2 D}} \right) \sqrt{x} - 1 = 0. \quad (A.8)
\]

Because \( x \geq 0 \) (\( r_1 \leq 1/\delta \)), we take the positive root of (A.7) to get
\[
 \sqrt{x} = \frac{\left( D \delta \sqrt{m_1 D} \sqrt{m_2 D} + \sqrt{m_1 D} \sqrt{m_2 D} - \sqrt{m_1 D} \sqrt{m_2 D} \right) + 4 - (D \delta \sqrt{m_1 D} \sqrt{m_2 D} + \sqrt{m_1 D} \sqrt{m_2 D} - \sqrt{m_1 D} \sqrt{m_2 D})}{2}. \quad (A.9)
\]

The expressions for \( r^*_1, r^*_2, \) and \( a^* \) can then be obtained from (A.8), (A.6), (A.3), and (2).
Proof of Proposition 2. We first prove the following lemma, which characterizes the best-response functions and the induced profit functions.

**Lemma 2.** For $i, j = 1, 2$ and $i \neq j$,

$$p_i'(p_i) = \begin{cases} 
    \frac{p_i + k_j^c - k_i^c - \frac{h}{r_j}}{r_i} & \text{if } p_i + k_j^c \geq c_i + k_i^c + \frac{2h}{r_i} + \frac{h}{r_j}, \\
    \frac{1}{2} \left( c_i - k_i^c + p_i + k_j^c + \frac{h}{r_j} \right) & \text{if } c_i + k_j^c - \frac{h}{r_j} \leq p_i + k_j^c \leq c_i + k_i^c + \frac{2h}{r_i} + \frac{h}{r_j}, \\
    \text{any } p_i \geq c_i, & \text{otherwise.}
\end{cases}$$

Proof of Lemma 2. For the best-response function $p_i'(p_i)$, we consider three cases depending on the value of $p_i$.

1. $p_i + k_j^c \geq c_i + k_i^c + \frac{2h}{r_i} + \frac{h}{r_j}$. Consider (7). Because supplier $i$ gets all the demand when $p_i \leq p_i + k_j^c - k_i^c + h/r_i$, $\Pi'(p_i)$

2. $0 \leq c_i + k_j^c - k_i^c \leq 2h/r_i + h/r_j$. Consider (7). Because supplier $i$ gets all the demand when $p_i \leq p_i + k_j^c - k_i^c + h/r_i$, $\Pi'(p_i)$
increases linearly in $p_i$ for $c_i \leq p_i \leq p_i + k_i^2 - k_i^2 - h/r_i$. It can easily be shown that $\Pi (p_i, p_j)$ is decreasing in $p_i$ for $p_i + k_i^2 - k_i^2 - h/r_i \leq p_i \leq p_i + k_i^2 - k_i^2 + h/r_i$, and $\Pi (p_i, p_j) = 0$ for $p_i \geq p_i + k_i^2 - k_i^2 + h/r_i$. Hence, $\Pi (p_i, p_j)$ is maximized at $p_i = p_i + k_i^2 - k_i^2 - h/r_i$.

Case 2. $c_i + k_i^2 - h/r_i \leq p_i + k_i^2 \leq c_i + k_i^2 + 2h/r_i + h/r_i$. As in Case 1, $\Pi (p_i, p_j)$ increases linearly in $p_i$ for $c_i \leq p_i \leq p_i + k_i^2 - k_i^2 - h/r_i$, and is zero for $p_i \geq p_i + k_i^2 - k_i^2 + h/r_i$. It can easily be shown that $\Pi (p_i, p_j)$ is increasing in $p_i$ for $p_i + k_i^2 - k_i^2 - h/r_i \leq p_i \leq (c_i - k_i^2 + p_i + k_i^2 + h/r_i)/2$, and decreasing in $p_i$ for $(c_i - k_i^2 + p_i + k_i^2 + h/r_i)/2 \leq p_i \leq p_i + k_i^2 - k_i^2 + h/r_i$. Therefore, $\Pi (p_i, p_j)$ is maximized at $p_i = (c_i - k_i^2 + p_i + k_i^2 + h/r_i)/2$.

Case 3. $p_i + k_i^2 \leq c_i + k_i^2 - h/r_i$. For any $p_i \geq c_i$, $p_i + k_i^2 - p_i - k_i^2 \leq -h/r_i$, and supplier $i$ will have zero demand share. Thus, $\Pi (p_i, p_j) = 0$ and supplier $i$ is indifferent with any $p_i \geq c_i$. Hence the result for $p_i^*(p_i)$. The result for $\Pi (p_i(p_i))$ can then be obtained by substituting $p_i^*(p_i)$ back into (7).

Now we are ready to prove Proposition 2. The best-response functions $p_i^*(p_j)$ and $p_j^*(p_i)$ are plotted in Figure 3.

It is straightforward to show that $p_i^*(p_j)$ and $p_j^*(p_i)$ intersect at $z_i$ when $c_i + k_i^2 - c_i - k_i^2 > 2h/r_i + h/r_i$, and at $z_i$ when $0 \leq c_i + k_i^2 - c_i - k_i^2 \leq 2h/r_i + h/r_i$, where $z_i$ and $z_j$ are given by

$$
\begin{align*}
    z_i &= \left( c_i + k_i^2 - k_i^2 - \frac{h}{r_i} \right), \\
    z_j &= \frac{1}{3} \left( 2c_i + \frac{h}{r_i} + c_i + \frac{2h}{r_i} + k_i^2 - k_i^2, 2c_i + \frac{h}{r_i} \right. \\
    &\quad \left. + c_i + \frac{2h}{r_i} + k_i^2 - k_i^2 \right).
\end{align*}
$$

Hence the result. □

Proof of Proposition 3. The results can be proved by differentiating the appropriate expressions given in Proposition 3. Details are omitted. □

References


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