Cooperative Advertising with Bilateral Participation in a Distribution Channel

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ABSTRACT

We study cooperative (co-op) advertising in a distribution channel with two channel members: an upstream manufacturer promoting its product through the national advertising and a downstream retailer advertising the product in its local market. Conventional marketing practice suggests that the manufacturer should share a portion of the retailer's advertising cost to achieve high channel efficiency, commonly referred to as the manufacturer’s participation in co-op advertising. While most existing literature focuses on the above unilateral participation strategy, we advocate a bilateral participation strategy where the retailer also shares a portion of the manufacturer's advertising cost. In particular, we show that a unilateral participation strategy is in general not capable of achieving channel coordination. In contrast, a bilateral participation strategy is able to do so with properly chosen participation rates. For a popular additive demand function, we compare several co-op advertising strategies with both unilateral and bilateral participation and lead to several useful managerial insights.
1. INTRODUCTION

Cooperative (co-op) advertising is a coordinated effort by all members in a distribution channel to increase the customer demand and the overall profits. In a typical distribution channel, the upstream member can be a manufacturer of certain product, who often time promotes its product via the national level advertising to build the long-term image or “brand equity” for the company. Meanwhile, the downstream channel member can be a retailer, who usually advertises the product in its local market to induce short-term purchase. Traditionally, co-op advertising is achieved with the upstream manufacturer sharing a portion of the downstream retailer’s advertising costs, commonly referred to as the manufacturer’s participation rate (Bergen and John 1997).

Co-op advertising gains increasing popularity in today’s marketing programs. According to Nagler (2006), the total US expenditure of co-op advertising in 2000 was estimated at $15 billion, nearly a four-fold increase in real terms in comparison to $900 million in 1970. We also see the same trend at the company level. Intel’s expenditure on co-op advertising grew from $800 million in 1999 to $1.5 billion in 2001 (Elkin 1999 and 2001). It is very common for the downstream retailers to pay for the major portion of their promotional budgets with the manufacturer’s supporting allowances. In the personal computer industry, IBM offers a 50-50 split of advertising costs with retailers, while Apple Computer pays 75% of the media costs (Brennan 1988). Appliance retailers receive more than 75% of their total advertising dollars from cooperative programs (Bovee and Arens 1986). GE’s budgets for local advertising are three times as high as its national advertising budget (Young and Greyser 1983). More recently, Nagler (2006) conducted a large-scale study of 2286 brands, finding that 61.5% of the retailers enjoy a
participation rate of 50% and 33.5% of them receive a full (100%) subsidy from the manufacturers.

High participation rates from manufacturers imply an asymmetric relationship between the manufacturers and the retailers in the distribution channel. Historically, the upstream manufacturer has been assumed to lead the distribution channel. As a result, most existing research focuses on unilateral participation where the upstream manufacturer subsidizes the retailer’s local advertising expenses. However, as we will show in this paper, unilateral participation in general does not lead to channel coordination. Besides, the relative negotiation power between manufacturers and retailers has changed dramatically in recent years. Large retailers such as Wal-Mart effectively use their market power to push for reduced prices from their suppliers (manufacturers) and increase their profit margin significantly (Lucas 1995). Procter & Gamble (P&G), a former undeniable leader in the distribution channel (Walton and Huey 1992), is now collaborating with Wal-Mart in many aspects, including demand monitoring, JIT delivery, and information sharing (Foley et al., 1996). It seems that a unilateral participation model no longer reflects the new partnership between the manufacturers and the retailers.

To reflect the symmetric partnership between manufacturers and retailers, in this paper we propose a bilateral participation model for the co-op advertising where the downstream retailer also shares a portion of the manufacturer's advertising cost. This paper makes the following contributions to the co-op advertising literature. First, we prove under quite general assumptions that a bilateral participation strategy is able to achieve full channel coordination with properly chosen participation rates. In contrast, the unilateral participation strategy in general cannot achieve channel coordination. Second,
for a popular additive demand function, we compare several co-op advertising strategies with both unilateral and bilateral participation. In particular, we evaluate the impact from several key factors of game structures, such as the game leader and the decision powers, on the channel performance. We obtain the following observations: (i) The bilateral participation strategy is a Pareto improvement over the best unilateral participation strategy in terms of both channel members' profits. (ii) The distribution channel benefits from choosing the participation rates exogenously instead of endogenously by the channel members for all game structures. (iii) Both channel members have incentives to lead the Stackelberg game. However, the channel performs better when it is led by the channel member with the higher profit margin.

The paper is organized as follows. Section 2 reviews both empirical and theoretical literature related to co-op advertising. Section 3 introduces a bilateral participation strategy for co-op advertising and shows its advantage over unilateral participation for the purpose of channel coordination. Section 4 evaluates and compares the channel performance under various game structures and participation strategies for a specific demand function. Managerial insights are drawn based on the quantitative analysis conducted. Section 5 concludes the paper and suggests potential future research topics.

2. LITERATURE REVIEW

Conventional marketing literature of co-op advertising often assumes an asymmetric relationship between channel members. The manufacturer, traditionally dominating the distribution channel, is assumed to be the channel leader. As a result, both empirical and analytical studies on co-op advertising focus on the unilateral participation strategy.
Among empirical studies, Lyon (1932), in his seminal work, initiates the investigations on the manufacturer’s advertising expenses and the net profit of vendor cooperative advertising allowances. Hutchins (1953) presents a real-world application of the manufacturer’s participation methodology and names co-op advertising for the manufacturer ‘the way to make it pay’. Through extensive empirical research, Nagler (2006) establishes a quadratic relationship between the manufacturer’s participation rates and the national advertising expenditure, i.e., the brand-level advertising investment.

Analytical models on co-op advertising typically examine how the channel performance is influenced by various factors, such as advertisement efforts at both the national and local levels, manufacturer’s participation rate, sales volume, etc. They are designed to capture the inherent interdependence and interactive conflict among channel members. Therefore, game theory has been a popular vehicle in analyzing co-op advertising.

Early studies assume that manufacturers’ advertising allowances are a function of the downstream retailer’s sales volume (e.g. Berger 1972). Berger and Magliozzi (1992) consider optimization of vertical co-operative promotion decisions in a direct mail operation. They show how the manufacturer and the direct mailer can cooperate and make decisions jointly to improve the channel profits. Dant and Berger (1996) extend the early work by Berger (1972) to the case where the sales function is probabilistic and the channel members have different opinions about the sales function. Later work measures manufacturers’ contribution to retailers’ local advertising by their participation rates (a percentage of retailers’ advertisement costs). For example, Bergen and John (1997) consider the impact of participation rates on advertising ‘spillover’. They conclude that
more generous participation rates are called for with less targetable media, less
differentiated retailers, more differentiated brands, and more upscale products within a
category.

Two major types of game-theoretic models have been developed for co-op advertising: static games and differential games. Static games are often used to characterize equilibrium solutions in the leader-follower and partnership environment, and they deal with the design of mechanisms (contracts) for the benefit of channel coordination. Dant and Berger (1996) address the vertical cooperative advertising decisions within a franchising context. They use sequential game models to demonstrate that cooperative determination of the franchisor’s and the franchisee’s advertising contributions may yield superior payoffs for both partners and result in optimal system-wide returns. Bergen and John (1997) focus on effects of co-op advertising across competing retailers and competing manufacturers through a static single-period model. Even though they make a distinction between “local” and “national” advertising, the demand function they use only implicitly considers the impact from the national advertisement. Huang and Li (2001), Li et al. (2002), Huang et al. (2002) introduce a specific demand function that explicitly captures the impact of national and local advertisements on the customer demand. They develop three game models to determine the manufacturer’s optimal participation rate and optimal advertisement efforts in a two-member distribution channel. Yue et al. (2006) extend the results of Huang and Li (2001) to the case where the manufacturer provides price discounts directly to its customers.

Differential games, on the other hand, prove to be powerful at modeling the
dynamics of the sales and channel members’ cost and profit changes over time. Chintagunta and Jain (1992) study a two-level channel in which both channel members advertise to increase their respective goodwill among consumers (ad’s carryover effect). Jorgensen and Zaccour (1999, 2003a, 2003b) conduct extensive studies under various games settings, including the presence of national advertising, participation rates, pricing, negative effect of advertising, etc.

All studies mentioned above, however, assume an asymmetric relationship between the channel members where the manufacturer is assumed to be the game leader. There have been a few exceptions in recent years as the retailers gain increasing power in the distribution channel. Huang and Li (2001) propose a bargaining procedure to determine the best co-op advertising scheme to achieve channel coordination. Buratto et al. (2007) study two vertical co-op advertising programs by alternatively assigning the manufacturer and the retailer as the game leader to make final decision on the participation rate. Our paper introduces bilateral participation into the co-op advertising environment to reflect a more symmetric relationship between the channel members.

3. BILATERAL PARTICIPATION AND CHANNEL COORDINATION

We consider the distribution channel of a product with one manufacturer \( M \) and one retailer \( R \). The demand of the product \( S(a_M, a_R) \) is a deterministic function of both the manufacturer’s national promotional effort \( a_M \) and the retailer’s local advertising expenditure \( a_R \). We assume that the manufacturer’s and retailer’s profit margins are \( \rho_M \) and \( \rho_R \), respectively. Denote \( r = \rho_M / \rho_R > 0 \) as the profit margin ratio, which reflects the relative bargaining power between the upstream manufacturer and the downstream
Consider the case of bilateral participation where the manufacturer shares $t_M \in [0,1]$ portion of the retailer’s local advertising cost $a_R$, and the retailer shares $t_R \in [0,1]$ portion of the manufacturer’s national promotion cost $a_M$. It is easy to verify that the profits for the manufacturer, the retailer, and the whole distribution channel are

$$\Pi_M(a_M, a_R; t_M, t_R) = \rho_M S(a_M, a_R) - (1-t_R) a_M - t_M a_R, \quad (1)$$

$$\Pi_R(a_M, a_R; t_M, t_R) = \rho_R S(a_M, a_R) - t_R a_M - (1-t_M) a_R, \quad (2)$$

$$\Pi(a_M, a_R) = \Pi_M + \Pi_R = \rho S(a_M, a_R) - a_M - a_R, \quad (3)$$

respectively, where $\rho = \rho_M + \rho_R$ is the profit margin of the distribution channel. Clearly, the bilateral participation reduces to the traditional unilateral participation when $t_R = 0$.

Suppose the manufacturer and the retailer play the Stackelberg game, with the manufacturer being the game leader, to determine their advertisement expenditures. Then the channel members act according to the following sequence:

i) The manufacturer and the retailer sign a contract, which specifies their respective participation rates $(t_M, t_R)$.

ii) They make sequential decisions on their respective advertisement expenditures.

More specifically, the manufacturer first decides its national promotion expenditure $a_M$, and the retailer then decides its local advertising expense $a_R$.

iii) Market demand $S(a_M, a_R)$ realizes. The manufacturer and the retailer obtain their profits according to equations (1) and (2), respectively.

We assume that the participation rates exogenously specified in step (i) will not change throughout the game and other parameters of the game are common knowledge to both channel members. The following result shows that a bilateral participation strategy,
with the participation rates properly specified, can lead to full channel coordination under some mild assumptions of the demand function (see Appendix A1 for a proof).

**Proposition 1.** Suppose the manufacturer $M$ and the retailer $R$ determine their advertisement expenditures according to the Stackelberg game specified above, with $M$ being the leader and $R$ the follower. If the demand function $S(a_M, a_R)$ is continuous, twice differentiable, and strictly concave, then the distribution channel is coordinated if and only if the bilateral participation rates are given by

$$t_M = r/(1+r) \text{ and } t_R = 1-t_M. \quad (4)$$

We have the following remarks based on Proposition 1:

**Remark 1.** The unilateral participation strategy is in general not able to coordinate the distribution channel. Since the unilateral participation strategy is a special case of the bilateral participation strategy with $t_R = 0$, Proposition 1 implies that unilateral participation achieves channel coordination if and only if $r \rightarrow \infty$ or $\rho_R = 0$, which happens only when the retailer makes zero profit. This, however, is unrealistic. In particular, the unilateral participation strategy considered in Huang and Li (2001) cannot coordinate the distribution channel, since their demand function $S(a_M, a_R) = \alpha - \beta a_R^{-\gamma} a_M^{-\delta}$, with positive parameters $\alpha, \beta, \gamma, \delta$, satisfies the assumptions of Proposition 1.

**Remark 2.** If the manufacturer’s and the retailer’s participation rates satisfy condition (4) in Proposition 1, their profit functions are proportional to the total channel profit:

$$\Pi_M = \rho_M S(a_M, a_R) - (\rho_M/\rho)a_M - (\rho_M/\rho)a_R = (r/(1+r))\Pi, \quad (5)$$

$$\Pi_R = \rho_R S(a_M, a_R) - (\rho_R/\rho)a_M - (\rho_R/\rho)a_R = (1/(1+r))\Pi. \quad (6)$$

As a result, both channel members’ optimal actions are consistent with the optimal action for the whole channel, which leads to channel coordination. In other words, proper
bilateral cost sharing aligns channel members’ interests with that of the distribution channel.

**Remark 3.** Proposition 1 remains true when the retailer \( R \), rather than the manufacturer, takes the lead in the Stackelberg game since bilateral participation ensures the symmetric relationship between the channel members. Similarly, the same results also hold when the manufacturer and the retailer play the Nash game, as long as the demand function \( S(a_M, a_R) \) is supermodular in \((a_M, a_R)\), which guarantees the existence of a Nash equilibrium. On the other hand, we show in Appendix A2 that the unilateral participation strategy fails to coordinate the distribution channel, even if the channel members play the Nash game.

**Remark 4.** For the bilateral participation strategy to coordinate the distribution channel, it is necessary that the participation rates \( t_M \) and \( t_R \) be exogenously specified according to condition (4). If channel members are given the right to specify their own participation rates endogenously, their choices will not be consistent with condition (4). In fact, if the retailer were to choose its participation rate \( t_R \), it would have chosen \( t_R = 0 \), since \( \frac{\partial \Pi_R}{\partial t_R} = -a_M < 0 \). Clearly, this leads to unilateral participation, which in general cannot coordinate the distribution channel.

### 4. EFFICIENCY EVALUATION

Section 3 proves theoretically that bilateral participation dominates unilateral participation in terms of channel coordination for co-op advertising. In this section, we will provide quantitative comparisons between these two participation strategies by assuming a specific demand function. In particular, our comparisons are conducted under
various game structures and settings specified by who leads the Stackelberg game, who makes the decisions on the participation rates as well as whose advertisement expenditure, and whether these decisions are made exogenously or endogenously.

4.1 Demand Function

There is a substantial amount of literature on the estimation of the demand function with respect to advertising spending. Some studies do not distinguish the impacts on sales between local (retailer) and national (manufacturer) advertising expenditures (Berger 1972; Little 1979; Tull et al. 1986; Dant and Berger 1996). Others consider their impacts on the sales separately (Jorgensen and Zaccour, 2000, 2003a and 2003b). We follow the latter approach by assuming the following demand function:

$$S(a_M, a_R) = k_M \sqrt{a_M} + k_R \sqrt{a_R},$$

(7)

where $k_M, k_R$ are positive constants reflecting the efficacy of each type of advertisement in generating sales. Notice that the demand function $S(a_M, a_R)$ is strictly increasing and concave with respect to $a_M$ and $a_R$, which reflects the commonly observed “advertising saturation effect,” i.e., additional advertising spending generates continuously diminishing returns. This property has been verified and supported by Simon and Arndt (1980) based on their review of over 100 studies. Denote $k = k_M^2 / k_R^2$. Clearly, $k$ reflects the relative effectiveness of national versus local advertisements in generating customer demands.

4.2 Who Leads the Game?

It is well known that the leader of a Stackelberg game gains a higher profit than its profit from the corresponding Nash game. This explains why both channel members have incentives to lead the game. Traditional literature in co-op advertising tends to let the
manufacturer lead the game ($M$ leads). Since the downstream retailer has been gaining bargaining power in recent years, as argued in our literature review, it is natural and necessary for us to also consider the co-op advertising environment with the retailer being the game leader ($R$ leads). This is reflected in our quantitative evaluations.

4.3 Who Makes Which Decision?

The equilibrium of the game as well as the resulting profit for each player depends on the allocation of decision power to each player. In fact, inappropriate allocation of decision powers can make the game ill defined. For example, for our co-op advertising model, if we let the retailer decide both the manufacturer’s participation rate $t_M$ and its own advertising effort $a_R$, the Stackelberg game will have a trivial and unreasonable outcome with $t_M = 1$ and $a_R = \infty$. In our quantitative comparison, we consider various logical combinations of decision power possibilities and evaluate the associated equilibriums. The participation rates $(t_M$, $t_R)$ can be either exogenously specified or endogenously determined. In the latter case, the $(t_M$, $t_R)$ pair should be determined by the game leader to rule out trivial cases. On the other hand, we always let the channel member determine its own advertisement expenditure except for a few necessary adjustments needed to avoid pathological situations (see footnotes of Table 1).

4.4 Benchmark Cases

To perform channel efficiency evaluations and comparisons, we introduce two benchmark cases: the Full co-op case and the Null co-op case. The former represents the fully coordinated distribution channel with all decisions centrally made; the latter describes the case where the channel members do not share each other’s advertisement costs and make their decisions independently. We will associate the over bar ‘$\bar{\hspace{1cm}}$’ with the
Full co-op case and the under bar '_' with the Null co-op case in our evaluation report.

In the Full co-op channel, both local and national advertisement expenditures are determined to optimize the overall channel profit (3) with \( S(a_M, a_R) \) given by (7). Since \( \Pi(a_M, a_R) \) is strictly joint concave in \((a_M, a_R)\), the optimal advertisement expenditures \( \bar{a}_r^* \) and \( \bar{a}_M^* \) can be obtained by the first order optimality conditions and \( \Pi^* = \Pi(\bar{a}_M^*, \bar{a}_R^*) \) represents the highest possible profit for the distribution channel.

In the Null co-op channel, there is no mutual cost sharing. The profit function for the manufacturer and the retailer are given by (1) and (2), respectively, with \( t_M = t_R = 0 \). In this situation, no matter who takes the lead, the optimal channel profit will be \( \Pi^* = \Pi(\bar{a}_M^*, \bar{a}_R^*) \), where \( \bar{a}_M^* (< \bar{a}_M^* ) \) and \( \bar{a}_R^* (< \bar{a}_R^* ) \) maximize (1) and (2) with \( t_M = t_R = 0 \), respectively.

4.5 Channel Efficiency Evaluations

The channel efficiency evaluations are summarized in Table 1 (with sample derivations given in Appendix A3). In particular, we consider three key factors that determine the structure and setting of the Stackelberg game for our co-op advertising model: the participation strategies (unilateral (Uni) or bilateral (Bi)), the decision power on \((t_M, t_R)\) (endogenously determined (En) or exogenously specified (Ex)), and game leaders (M or R). We consider game settings from all logical combinations of the above three key factors and denote each game setting by “Uni(Bi)/En(Ex)/M(R)”. For each game setting, we evaluate the manufacturer’s and retailer’s optimal decisions (participation rates \((t_M^*, t_R^*)\) and advertising expenditures \((a_M^*, a_R^*)\) ) at the equilibrium as well as the associated profits for each channel members \((\Pi_M^*, \Pi_R^*)\) and for the whole distribution channel \( \Pi^* \). Furthermore, we introduce the ratio \( \eta = \Pi^*/\bar{\Pi}^* \in (0,1) \) to measure the
channel efficiency for a given game setting. For example, we have $5<\eta<\frac{8}{9}$ in Table 1, indicating that the channel profit from Null co-op can be as low as 56% of that of the coordinated channel ($\Pi^*$) and is limited to no more than 88% of $\Pi^*$. Here we should notice that many of the channel performance derivations and efficiency evaluations require the ratio $r$ to stay in the range $\left(\frac{1}{2}, 2\right)$ so as to rule out pathological situations or trivial discussions in which the participation rate(s) may fall out of range $(0,1)$.

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Insert Table 1 here

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4.6 Managerial Insights

Careful studying of Table 1 and direct comparisons allow us to draw the following managerial insights:

**Insight 1.** For both unilateral and bilateral participation, the channel achieves higher profits when the participation rates are optimally specified exogenously than when they are determined endogenously, no matter who leads the distribution channel.

**Insight 2.** For both unilateral and bilateral participation strategies, if the participation rates are endogenously chosen, then the channel member with a higher profit margin should lead the channel to improve channel efficiency. If the participation rates are optimally specified exogenously, it does not matter who leads the channel.

The second portion of Insight 2 is a direct observation from Table 1. The first portion can be verified by comparing the channel profits for cases with $r>1$ ($\rho_M > \rho_R$) and $r<1$ ($\rho_M < \rho_R$) as the manufacturer and the retailer take the lead.

**Insight 3.** No matter how the participation rates are determined, exogenously or
endogenously, and no matter who leads the channel, the manufacturer or the retailer, bilateral participation leads to higher channel profits than unilateral participation.

Both theoretical and quantitative results in the paper suggest that the bilateral participation strategy is better than the unilateral participation strategy for the co-op advertising in terms of the channel profit. However, it is not clear if the channel members have the incentive to move from the unilateral participation strategy to the bilateral participation strategy. Our last less obvious insight provides a positive answer to this question. The detailed proofs are given in Appendix A4.

**Insight 4.** For any unilateral participation strategy with the participation rate either endogenously or exogenously determined, there exists a Pareto improvement bilateral participation strategy to make both channel members better off in terms of their profits.

### 5. CONCLUSIONS AND FUTURE RESEARCH

Even though unilateral participation is a common practice in current co-op advertising, the paper introduces and advocates the concept of bilateral participation. It is shown, through both theoretical analysis and quantitative evaluation, that bilateral participation is a Pareto improvement over unilateral participation for all logical distribution channel settings (channel leader and decision power). In addition, our research suggests that for both unilateral and bilateral participation, the participation rates should be determined exogenously, and the distribution channel should be led by the member with the higher profit margin to maximize the channel efficiency.

For future research, we would like to see hypothesis testing of the above managerial insights with industrial data. Extending the distribution channel to multiple retailers
and/or multiple manufacturers may help explore other dimensions of channel coordination, such as price competition, product differentiation, and channel decentralization, among others. These issues were studied in the field of pricing (Choi 1996; Ingene and Parry 1995a, 1995b, 1998, 2000) but rarely in co-op advertising. The role of asymmetric information in co-op advertising is also an interesting area for future research.
REFERENCES


Appendices
A1. Proof of Proposition 1

Necessity: For the centralized system, the profit for the whole is given by (3). Since the market demand $S(a_M, a_R)$ is strictly concave in $(a_M, a_R)$, there is a unique solution $(\tilde{a}_M^*, \tilde{a}_R^*)$ to the system’s optimality condition $\partial \Pi / \partial a_M = 0$ and $\partial \Pi / \partial a_R = 0$, which requires

\[ \frac{\partial S(a_M, a_R)}{\partial a_M} / \frac{\partial a_M}{\partial} = 1 / \rho , \tag{a1} \]

\[ \frac{\partial S(a_M, a_R)}{\partial a_R} / \frac{\partial a_R}{\partial} = 1 / \rho . \tag{a2} \]

The system achieves its optimum if and only if $(a_M^*, a_R^*) = (\tilde{a}_M^*, \tilde{a}_R^*)$.

However, the payoff functions of $M$ and $R$ are given by (1) and (2) respectively. When the two players are involved in a Stackelberg game with the manufacturer $M$ being the leader, for any given $a_M$, the retailer $R$ will choose its best response function $a_R(a_M; t_M, t_R)$ by solving $\partial \Pi / \partial a_R = 0$, which gives

\[ \rho_R \frac{\partial S(a_M, a_R)}{\partial a_R} / \frac{\partial a_R}{\partial} = 1 - t_M . \tag{a3} \]

Comparing (a3) with (a2), we have

\[ t_M = 1 - \rho_R / \rho = \rho_M / \rho = r/(1+r) . \tag{a4} \]

At the first stage of the game, $M$ will take $a_R(a_M; t_M, t_R)$ into account and maximize its profit function $\Pi_M(a_M, a_R; t_M, t_R) = \rho_M S(a_M, a_R(a_M; t_M, t_R)) - t_M a_R(a_M; t_M, t_R) - (1-t_R)a_M$. The first order condition on $a_M$ gives

\[ \rho_M \left[ \frac{\partial S(a_M, a_R)}{\partial a_M} + \frac{\partial S(a_M, a_R)}{\partial a_R} \frac{\partial a_R(a_M, t_M, t_R)}{\partial a_M} \right] = t_M \frac{\partial a_R(a_M, t_M, t_R)}{\partial a_M} + (1-t_R) . \tag{a5} \]

Utilizing conditions (a3) and (a4), we can simplify (a5) and obtain

\[ \rho_M \frac{\partial S(a_M, a_R)}{\partial a_M} / \frac{\partial a_M}{\partial} = 1 - t_R . \tag{a6} \]

Comparing (a6) with (a1), we have

\[ t_R = 1 - \rho_M / \rho = 1/(1+r) = 1 - t_M . \tag{a7} \]

Sufficiency: Under condition (4), it is easy to verify that the manufacturer’s and the retailer’s profits satisfy equations (5) and (6), respectively. In other words, they are proportional to the profit of
the whole distribution channel. Since $S(a_M, a_R)$ is strictly concave in $(a_M, a_R)$, the unique Stackelberg equilibrium will maximize the whole channel’s profit function. ■

A2. Proof for the Nash game case under unilateral participation

For the centralized system, the profit for the whole channel is given by (3), and the unique optimal advertising level $(a_M^*, a_R^*)$ solves (a1) and (a2). When the two members play a Nash game under unilateral participation, they will make decisions simultaneously with their profit functions specified by (1) and (2) respectively, with $t_R = 0$. For any exogenously given $t_M$, the members’ best response functions $a_M(a_R, t_M)$ and $a_R(a_M, t_M)$ solve $\partial \Pi_M / \partial a_M = 0$ and $\partial \Pi_R / \partial a_R = 0$, respectively, which gives

$$\rho_M \frac{\partial S(a_M, a_R)}{\partial a_M} = 1,$$  \hspace{1cm} (a8)

$$\rho_R \frac{\partial S(a_M, a_R)}{\partial a_R} = 1 - t_M.$$  \hspace{1cm} (a9)

Comparison of (a1), (a2) and (a8), (a9) reveals that the channel coordination is achieved only if $\rho_R = 0, t_M = 1$, which implies that the retailer’s profit vanishes. Therefore, unilateral participation is in general not able to coordinate the distribution channel. ■

A3. Derivations of results in Table 1

The derivation processes dealing with all the game settings listed in Table 1 are quite similar. To save space, we show only the detailed derivation for the cases with bilateral participation. For the cases with unilateral participation, we can use the similar approach by setting $t_R = 0$.

The Bi/En/M case:

Under this game setting, the manufacturer $M$ (the game leader) determines $(t_M, t_R)$ while the retailer $R$ (the game follower) determines $(a_M, a_R)$. For any given $(t_M, t_R)$, $R$ chooses its best response functions $a_M(t_M, t_R)$ and $a_R(t_M, t_R)$ such that $\partial \Pi_R / \partial a_M = 0$ and $\partial \Pi_R / \partial a_R = 0$ at the second stage of the game. As a result, we have

$$a_M(t_M, t_R) = \left( \frac{\rho_M k_M}{2t_R} \right)^2,$$  \hspace{1cm} (a10)
Substituting (a10) and (a11) into equation (1) and drawing the first order conditions of $\Pi_M$ on $(t_M, t_R)$, we get the optimal participation rates $t'_M = \frac{2r - 1}{2r + 1}$ and $t'_R = \frac{2}{2r + 1}$, where $r = \rho_M/\rho_R > 1/2$. Therefore, the optimal participation rates (a10) and (a11) can be expressed as

$$a^*_M = a_M(t'_M, t'_R) = \left[\frac{\rho_M k_M(2r+1)}{4r}\right]^2,$$

$$a^*_R = a_R(t'_M, t'_R) = \left[\frac{\rho_M k_R(2r+1)}{4}\right]^2,$$

and $M, R$ and the channel’s optimal profits (1), (2), (3) are

$$\Pi'_M = \frac{\rho_M^2 k_M^2}{4}(1+k)(r+1/2)^2,$$

$$\Pi'_R = \frac{\rho_M^2 k_R^2}{4}(1+k)(1+1/2),$$

$$\Pi^* = \frac{\rho_R^2 k_R^2}{4}[(1+r)^2-1/4](1+k).$$

The channel efficiency with $1/2 < r < 2$ is given by

$$\eta = \Pi^*/\Pi^* = \frac{(1+r)^2-1/4}{(1+r)^2}.$$ 

Since it is increasing in $r$, we can simply set $r \to 1/2$ and $r \to 2$ to obtain the lower bound $8/9$ and the upper bound $35/36$, respectively. ■

The Bi/En/R case:

The derivation for Bi/En/R case is very similar to the Bi/En/M case, except that now the manufacturer $M$ on the second stage of the game determines $(a_M, a_R)$, and the retailer $R$ determines both $t_M$ and $t_R$. Simple calculations show that the optimal participation rates are $t'_M = \frac{2r}{2+r}$ and $t'_R = \frac{2-r}{2+r}$, where $r = \rho_M/\rho_R < 2$ and the channel efficiency is given by

$$\eta = \Pi^*/\Pi^* = \frac{(1+r)^2-r^2/4}{(1+r)^2},$$

(18)
which achieves the minimum \((8/9)\) as \(r \to 2\), and achieves the maximum \((35/36)\) as \(r \to 1/2\).

The Bi/Ex/M(R) case:

As proved in Proposition 1, the distribution channel is fully coordinated with the optimal exogenously specified participation rates \(t^*_M = r/(1 + r)\) and \(t^*_R = 1/(1 + r)\), no matter who takes the lead. Therefore, the channel efficiency is 1. Under this situation, the optimal advertising expenditures, the channel members’ and the channel-wide optimal profits can be calculated accordingly.

A4. Proof of Insight 4

We are going to prove that for any given unilateral participation strategy with game settings specified by Uni/Ex(En)/M(R), we can always find a bilateral participation strategy which generates higher profits for both channel members under each one of the respective game settings. Here we only deal with the Uni/Ex/M case, since the similar methodology also applies to the cases of Uni/En/M and Uni/Ex/M(R).

For the Uni/Ex/M case, as shown in Table 1, the manufacturer \(M\)'s optimal participation rate is \(t^*_M = r/(1 + r)\) and the two members’ optimal profits are

\[
\Pi^*_M_{\text{Uni/Ex/M}} = \frac{\rho^2 k^2}{4} \left[ r(1 + r) + kr^2 \right]
\]

and

\[
\Pi^*_R_{\text{Uni/Ex/M}} = \frac{\rho^2 k^2}{4} \left[ (1 + r) + 2kr \right].
\]

Now we turn to the corresponding Bi/Ex/M case to find a Pareto improvement.

For each given pair \((t_M, t_R)\), the two channel members’ best response functions under Bi/Ex/M case will be

\[
a_M(t_M, t_R) = \left[ \frac{\rho^2 k^2}{2(1-t_M)} \right]^2
\]

and

\[
a_R(t_M, t_R) = \left[ \frac{\rho^2 k^2}{2(1-t_R)} \right]^2.
\]

Therefore, their optimal profits (1) and (2) can be expressed in terms of \(t_M\) and \(t_R\) as

\[
\Pi_M(t_M, t_R) = \frac{\rho^2 k^2}{4} \left[ \frac{2r}{1-t_M} - \frac{t_M}{(1-t_M)^2} + \frac{kr^2}{1-t_R} \right],
\]

(a19)

\[
\Pi_R(t_M, t_R) = \frac{\rho^2 k^2}{4} \left[ \frac{1}{1-t_M} + \frac{2kr}{1-t_R} - \frac{t_M kr^2}{(1-t_R)^2} \right].
\]

(a20)

To find a Pareto improvement policy, we need only to show that there exist \(\hat{t}_M, \hat{t}_R \in (0,1)\) such
that
\[
\Pi_M (\hat{t}_M, \hat{t}_R) > \Pi_M^* \bigg|_{Unil/Ex/M},
\]
\[
\Pi_R (\hat{t}_M, \hat{t}_R) > \Pi_R^* \bigg|_{Unil/Ex/M}.
\]

Since \( \Pi_M (t_M^*, 0) = \Pi_M^* \bigg|_{Unil/Ex/M} \) and \( \Pi_R (t_M^*, 0) = \Pi_R^* \bigg|_{Unil/Ex/M} \), we could simply choose
\[\hat{t}_M = t_M^* = \frac{r}{1 + r}, \]
and then we need only to prove that
\[\partial \Pi_M (t_M^*, t_R) / \partial t_R \bigg|_{t_R = 0} > 0,\]  
\[\partial \Pi_R (t_M^*, t_R) / \partial t_R \bigg|_{t_R = 0} > 0.\]

For fixed \( t_M = t_M^* = \frac{r}{1 + r} \), the derivatives of (a19) and (a20) with respect to \( t_R \) at \( t_R = 0 \) with \( r \) in \((1/2, 2)\) are
\[\partial \Pi_M (t_M^*, t_R) \bigg|_{t_R = 0} = \rho \frac{k^2}{4} \frac{kr^2}{(1 - t_R)^2} \bigg|_{t_R = 0} = \rho \frac{k^2}{4} kr^2 > 0,\]
\[\partial \Pi_R (t_M^*, t_R) \bigg|_{t_R = 0} = \rho \frac{k^2}{4} \frac{kr}{(1 - t_R)^2} \bigg|_{t_R = 0} = \rho \frac{k^2}{4} kr (2 - r) > 0.\]

This implies that there does exist some \( \hat{t}_R \in (0, 1) \) such that (a23) and (a24) hold. In other words, we are able to find a local improvement in terms of both members’ profits when switching from unilateral participation to bilateral participation.
Table 1. Measurements for channel performance and efficiency under different game settings

<table>
<thead>
<tr>
<th>Participation strategy</th>
<th>Decision powers on participation rate(s)</th>
<th>Game leader</th>
<th>Optimal channel profit ( \Pi^* )</th>
<th>Manufacturer’s profit ( \Pi^*_{a_M} )</th>
<th>Retailer’s profit ( \Pi^*_{a_R} )</th>
<th>National Adv. ( a_u )</th>
<th>Local Adv. ( a_s )</th>
<th>Participation rate(s)</th>
<th>System efficiency ( \eta = \Pi^* / \Pi^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full</strong></td>
<td>-</td>
<td>-</td>
<td>( \frac{\rho_k z^2}{4} (1 + r)^2 (1 + k) )</td>
<td>( \frac{\rho_k z^2}{4} r (1 + r) (1 + k) )</td>
<td>( \frac{\rho_k z^2}{4} (1 + r) (1 + k) )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>-</td>
<td>( \eta = 1 )</td>
</tr>
<tr>
<td><strong>Null</strong></td>
<td>-</td>
<td>M(R)</td>
<td>( \frac{\rho_k z^2}{4} [1 + 2r + k(2r + r^2)] )</td>
<td>( \frac{\rho_k z^2}{4} r (2 + rk) )</td>
<td>( \frac{\rho_k z^2}{4} [1 + 2r + k(2r + r^2)] )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>-</td>
<td>( 5 \leq \eta &lt; 1 )</td>
</tr>
<tr>
<td><strong>Uni</strong></td>
<td>En</td>
<td>M</td>
<td>( \frac{\rho_k z^2}{4} [(1 + r)^2 - 1/4 + k(2r + r^2)] )</td>
<td>( \frac{\rho_k z^2}{4} [(r + 1/2)^2 + kr^2] )</td>
<td>( \frac{\rho_k z^2}{4} [2rk + r + 1/2] )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{2r - 1}{2r + 1} )</td>
<td>( 5 \leq \eta &lt; 35/36 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R</td>
<td>( \frac{\rho_k z^2}{4} [(1 + r)^2 - r^2 / 4 + k(2r + r^2)] )</td>
<td>( \frac{\rho_k z^2}{4} [r^2 + (k + 1/2)^2 + 2rk] )</td>
<td>( \frac{\rho_k z^2}{4} [1 + r(1/2)^2 + 2kr] )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{2r}{2 + r} )</td>
<td>( 5 \leq \eta &lt; 35/36 )</td>
</tr>
<tr>
<td><strong>Ex</strong></td>
<td>M(R)</td>
<td>En</td>
<td>( \frac{\rho_k z^2}{4} [(1 + r)^2 + k(2r + r^2)] )</td>
<td>( \frac{\rho_k z^2}{4} [r(1 + r) + 2kr] )</td>
<td>( \frac{\rho_k z^2}{4} [1 + r + 2k] )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{r}{1 + r} )</td>
<td>( 5 \leq \eta &lt; 1 )</td>
</tr>
<tr>
<td><strong>Bi</strong></td>
<td>En</td>
<td>M</td>
<td>( \frac{\rho_k z^2}{4} [(1 + r)^2 - 1/4] + k) )</td>
<td>( \frac{\rho_k z^2}{4} [(1 + k)(r + 1/2)^2] )</td>
<td>( \frac{\rho_k z^2}{4} [1 + k + 1/2] )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{2r - 1}{2r + 1} )</td>
<td>( 8 &lt; \eta &lt; 35/36 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R</td>
<td>( \frac{\rho_k z^2}{4} [(1 + r)^2 - r^2 / 4] + k) )</td>
<td>( \frac{\rho_k z^2}{4} [(1 + k)(r + r^2 + 2) / 2] )</td>
<td>( \frac{\rho_k z^2}{4} [1 + k + 1/2 + 2r] )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{2r - 1}{2r + 1} )</td>
<td>( 8 &lt; \eta &lt; 35/36 )</td>
</tr>
<tr>
<td><strong>Ex</strong></td>
<td>M(R)</td>
<td>En</td>
<td>( \frac{\rho_k z^2}{4} (1 + r)^2 (1 + k) )</td>
<td>( \frac{\rho_k z^2}{4} (1 + r)^2 (1 + k) )</td>
<td>( \frac{\rho_k z^2}{4} (1 + r)^2 (1 + k) )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{\rho_k z^2}{2} )</td>
<td>( \frac{r}{1 + r} )</td>
<td>( 1 \leq \eta = 1 )</td>
</tr>
</tbody>
</table>

a): For cases Uni/En/R, Bi/En/M and Bi/En/R, decision powers are rearranged to avoid trivial and illogical game solutions as follows: for the Uni/En/R case, M determines \((a_{M_U}, a_R)\) and R determines \(\epsilon_{M_U}\); for the Bi/En/M case, M determines \((\epsilon_{M_U}, \epsilon_{M_B})\) and R determines \((a_{M_U}, a_R)\); for the Bi/En/R case, M determines \((a_{M_U}, a_R)\) and R determines \((\epsilon_{M_U}, \epsilon_{M_B})\).