Second eigenvalue of a Jacobi operator of hypersurfaces with constant scalar curvature

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Abstract

Let \( x : M \to \mathbb{S}^{n+1}(1) \) be an \( n \)-dimensional compact hypersurface with constant scalar curvature \( n(n-1)r, \ r \geq 1 \), in a unit sphere \( \mathbb{S}^{n+1}(1) \), \( n \geq 5 \) and \( J_s \) be the Jacobi operator of \( M \). In [7], Q. -M. Cheng studied the first eigenvalue of the Jacobi operator \( J_s \) of the hypersurface with constant scalar curvature \( n(n-1)r, \ r > 1 \) in \( \mathbb{S}^{n+1}(1) \). In [2], L. J. Alías, A. Brasil and L. A. M. Sousa studied the first eigenvalue of \( J_s \) of the hypersurface with constant scalar curvature \( n(n-1) \) in \( \mathbb{S}^{n+1}(1) \), \( n \geq 3 \). In this paper, we study the second eigenvalue of the Jacobi operator \( J_s \) of \( M \) and give an optimal upper bound for the second eigenvalue of \( J_s \).

2000 Mathematics Subject Classification: Primary 53C42; Secondary 58J50.

Key words and phrases: hypersurface with constant scalar curvature, second eigenvalue, Jacobi operator, mean curvature, principal curvature.

1 Introduction

Let \( M \) be an \( n \)-dimensional compact hypersurface in a unit sphere \( \mathbb{S}^{n+1}(1) \). We denote the components of the second fundamental form of \( M \) by \( h_{ij} \), and the principal curvatures of \( M \) by \( k_1, \ldots, k_n \). Let \( H, H_2 \) and \( H_3 \) denote the mean curvature, the 2nd mean curvature and the 3rd mean curvature of \( M \) respectively, namely,

\[
H = \frac{1}{n} \sum_{i=1}^{n} k_i,
\]

\[
H_2 = \frac{2}{n(n-1)} \sum_{1 \leq i_1 < i_2 \leq n} k_{i_1} k_{i_2},
\]

\[
H_3 = \frac{6}{n(n-1)(n-2)} \sum_{1 \leq i_1 < i_2 < i_3 \leq n} k_{i_1} k_{i_2} k_{i_3}.
\]

We denote the square norm of the second fundamental form of \( M \) by \( S \). A Schrödinger operator

\[
J_m = -\Delta - S - n,
\]

∗Supported by NSFC grant No. 10971110 and Tsinghua University–K.U.Leuven Bilateral scientific cooperation Fund.
†Supported by NSFC grant No. 10701007 and Tsinghua University–K.U.Leuven Bilateral scientific cooperation Fund.
where \( \Delta \) stands for the Laplace-Beltrami operator, is the Jacobi operator, whose spectral behavior is directly related to the instability of both the minimal hypersurfaces and the hypersurfaces with constant mean curvature in \( S^{n+1}(1) \) (cf. [19] and [3]). The first eigenvalue of the Jacobi operator \( J_m \) of such hypersurfaces in \( S^{n+1}(1) \) was studied by Simons [19] and Wu [22].

The second eigenvalue of the Jacobi operator \( J_m \) of the compact hypersurface in \( S^{n+1}(1) \) was studied by A. El Soufi and S. Ilias in [20]. They obtained that if \( M \) is an \( n \)-dimensional compact hypersurface in \( S^{n+1}(1) \), then the second eigenvalue \( \lambda_{2}^{J_m} \) of the Jacobi operator \( J_m \) satisfies

\[
\lambda_{2}^{J_m} \leq 0,
\]

where the equality holds if and only if \( M \) is a totally umbilical hypersurface in \( S^{n+1}(1) \).

For any \( C^2 \)-function \( f \) on \( M \), we define a differential operator

\[
\Box f = \sum_{i,j=1}^{n} (nH\delta_{ij} - h_{ij}) f_{ij},
\]

(1.1)

where \( (f_{ij}) \) is the Hessian of \( f \). We notice that the differential operator \( \Box \) is self-adjoint and it was introduced by S.Y.Cheng and Yau in [8] in order to study the compact hypersurfaces with constant scalar curvature in \( S^{n+1}(1) \). They proved that if \( M \) is an \( n \)-dimensional compact hypersurface with constant scalar curvature \( n(n - 1)r, \ r \geq 1 \), and if the sectional curvature of \( M \) is non-negative, then \( M \) is either a totally umbilical hypersurface \( S^m(c) \) or a Riemannian product \( S^m(c) \times \mathbb{S}^{n-m}(\sqrt{1 - c^2}) \), \( 1 \leq m \leq n - 1 \), where \( S^k(c) \) denotes a sphere of radius \( c \).

In [12], the author proved that if \( M \) is an \( n \)-dimensional \( (n \geq 3) \) compact hypersurface with constant scalar curvature \( n(n - 1)r, \ r \geq 1 \), and if \( S \leq (n - 1)\frac{n(r-1)+2}{n-2} + \frac{n-2}{r(n-1)+2} \), then \( M \) is either a totally umbilical hypersurface or a Riemannian product \( S^1(c) \times \mathbb{S}^{n-1}(\sqrt{1 - c^2}) \) with \( 0 < 1 - c^2 = \frac{n-2}{nr} \leq \frac{n-2}{n} \). Furthermore, the Riemannian product \( S^1(c) \times \mathbb{S}^{n-1}(\sqrt{1 - c^2}) \) has been characterized in [5] and [9].

In [1], Alencar, do Carmo and Colares studied the stability of hypersurfaces with constant scalar curvature in \( S^{n+1}(1) \). In this case, the Jacobi operator \( J_s \) is given by (cf. [1] and [7])

\[
J_s = -\Box - \{n(n - 1)H + nHS - f_3\},
\]

(1.2)

which is associated with the variational characterization of hypersurfaces with constant scalar curvature in \( S^{n+1}(1) \), where \( f_3 = \sum_{j=1}^{n} k^3_j \) (cf. [17] and [18]). The spectral behavior of \( J_s \) is directly related to the instability of the hypersurfaces with constant scalar curvature.

In general, \( J_s \) is not an elliptic operator. When \( r > 1 \), the differential operator \( \Box \) and hence \( J_s \) is an elliptic operator (cf. pages 3310, 3311 in [7]). When \( r = 1 \), if we assume that \( H_3 \neq 0 \) on \( M \), then we get \( J_s \) is elliptic (cf. Proposition 1.5 in [11]).

**Definition 1:** We call \( \lambda_{J_s}^r \) an eigenvalue of \( J_s \) if there exists a non-zero function \( f \) on \( M \) such that \( J_s f = \lambda_{J_s}^r f \), we call \( \lambda_{\Box}^r \) an eigenvalue of \( \Box \) if there exists a non-zero function \( f \) on \( M \) such that \( \Box f + \lambda_{\Box}^r f = 0 \), and we call \( \lambda_{\Delta}^r \) an eigenvalue of \( \Delta \) if there exists a non-zero function \( f \) on \( M \) such that \( \Delta f + \lambda_{\Delta}^r f = 0 \).

In [7], Q.-M. Cheng studied the first eigenvalue of \( J_s \) of the hypersurface \( M \) with constant scalar curvature \( n(n - 1)r, \ r > 1 \) in \( S^{n+1}(1) \), and gave an optimal upper bound for the first eigenvalue of \( J_s \).
Theorem 1.1. (see Corollary 1.2 in [7]) Let $M$ be an $n$-dimensional compact orientable hypersurface with constant scalar curvature $n(n - 1)r$, $r > 1$, in $S^{n+1}(1)$. Then the Jacobi operator $J_s$ is elliptic and the first eigenvalue of $J_s$ satisfies

$$
\lambda_{1s} \leq -n(n - 1)r\sqrt{r - 1},
$$

where the equality holds if and only if $M$ is totally umbilical and non-totally geodesic.

In [2], L. J. Alí as, A. Brasil and L. A. M. Sousa studied the first eigenvalue $\lambda_{1s}$ of $J_s$ of the hypersurface $M$ with constant scalar curvature $n(n - 1)$ in $S^{n+1}(1)$.

Theorem 1.2. ([2]) Let $M$ be an $n$-dimensional compact orientable hypersurface with constant scalar curvature $n(n - 1)$, in $S^{n+1}(1)$, $n \geq 3$. Assume that $H_3 \neq 0$, then the Jacobi operator $J_s$ is elliptic and the first eigenvalue $\lambda_{1s}$ of the Jacobi operator $J_s$ satisfies

$$
\lambda_{1s} \leq -2n(n - 1) \min |H|
$$

and the equality holds if and only if $M$ is the Riemannian product $S^m(c) \times S^{n-m}(\sqrt{1-c^2})$, $1 \leq m \leq n - 2$, $c = \sqrt{(n-1)m+\sqrt{(n-1)m(n-m)}}/n(n-1)$ in $S^{n+1}(1)$.

In this paper, we study the second eigenvalue for $J_s$ of the hypersurface $M$ with constant scalar curvature $n(n - 1)r$, $r \geq 1$ in $S^{n+1}(1)$, $n \geq 5$, and we have the following results.

Theorem 1.3. Let $M$ be an $n$-dimensional compact orientable hypersurface with constant scalar curvature $n(n - 1)r$, $r \geq 1$ in $S^{n+1}(1)$, $n \geq 5$. Then, the Jacobi operator $J_s$ is elliptic and the second eigenvalue $\lambda_{2s}$ of the Jacobi operator $J_s$ satisfies

$$
\lambda_{2s} \leq 0
$$

and the equality holds if and only if $M$ is totally umbilical and non-totally geodesic.

Theorem 1.4. Let $M$ be an $n$-dimensional compact orientable hypersurface with constant scalar curvature $n(n - 1)$, in $S^{n+1}(1)$, $n \geq 5$. Assume that $H_3 \neq 0$, then the Jacobi operator $J_s$ is elliptic and the second eigenvalue $\lambda_{2s}$ of the Jacobi operator $J_s$ satisfies

$$
\lambda_{2s} \leq -\frac{n(n - 1)(n - 2)}{2} \min |H_3| \quad (1.3)
$$

and the equality holds if and only if $H_3 = \text{constant} \neq 0$ and the position functions of $M$ in $S^{n+1}(1)$ are the second eigenfunctions of $J_s$ corresponding to $\lambda_{2s}$. In particular, when $M$ is the Riemannian product $S^m(c) \times S^{n-m}(\sqrt{1-c^2})$, $1 \leq m \leq n - 2$, $c = \sqrt{(n-1)m+\sqrt{(n-1)m(n-m)}}/n(n-1)$ in $S^{n+1}(1)$, the equality in (1.3) is attained.

2 Preliminaries

Throughout this paper, all manifolds are assumed to be smooth and connected without boundary. In this section, we give some formulas and notations of hypersurfaces in a unit sphere by
using the method of moving frames. Let \( x : M \to S^{n+1}(1) \) be an n-dimensional hypersurface in a unit sphere \( S^{n+1}(1) \). We make the following convention on the range of indices:

\[ 1 \leq i, j, k, l \leq n. \]

Let \( \{e_1, \cdots, e_n, e_{n+1}\} \) be a local orthonormal basis of \( T S^{n+1}(1) \) with dual basis \( \{\omega_1, \cdots, \omega_n, \omega_{n+1}\} \) such that when restricted on \( M \), \( \{e_1, \cdots, e_n\} \) is a local orthonormal basis of \( TM \). Hence we have \( \omega_{n+1} = 0 \) on \( M \) and we have the following structure equations (see [4], [9], [12] and [19]):

\[
dx = \sum_i \omega_i e_i, \tag{2.1}\]

\[
de_i = \sum_j \omega_{ij} e_j + \sum_j h_{ij} \omega_{e_{n+1}} - \omega_i x, \tag{2.2}\]

\[
de_{n+1} = - \sum_{i,j} h_{ij} \omega_i e_i, \tag{2.3}\]

where \( h_{ij} \) denote the components of the second fundamental form of \( M \).

The Gauss equations are (see [9], [12])

\[
R_{ijkl} = \delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk} + h_{ik} h_{jl} - h_{il} h_{jk}, \tag{2.4}\]

\[
R_{ik} = (n - 1) \delta_{ik} + n H h_{ik} - \sum_j h_{ij} h_{jk}, \tag{2.5}\]

\[
R = n(n - 1) r = n(n - 1) + n^2 H^2 - S, \tag{2.6}\]

where \( R \) is the scalar curvature of \( M \), \( r \) is the normalized scalar curvature of \( M \) and \( S = \sum_{i,j} h_{ij}^2 \) is the norm square of the second fundamental form, \( H = \frac{1}{n} \sum_i h_{ii} \) is the mean curvature of \( M \).

The Codazzi equations are given by (see [9], [12])

\[
h_{ijk} = h_{ikj}. \tag{2.7}\]

Let \( f \) be a smooth function on \( M \), we define the gradient and Hessian by (see [9], [12])

\[
df = \sum_{i=1}^{n} f_i \omega_i, \tag{2.8}\]

\[
\sum_{i=1}^{n} f_i \omega_j = df + \sum_{j=1}^{n} f_j \omega_j. \tag{2.9}\]

Then the Jacobi operator \( J_s f \) (see [12]) is defined by

\[
J_s f = -\Box f - (n(n - 1) H + n H S - f_3) f
= -\sum_{i,j} (n H \delta_{ij} - h_{ij}) f_{ij} - (n(n - 1) H + n H S - f_3) f. \tag{2.10}\]
3 Some examples and some lemmas

First of all, we consider the first and second eigenvalues of the Jacobi operator $J_s$ of the totally umbilical and non-totally geodesic hypersurface in $\mathbb{S}^{n+1}(1)$ with constant scalar curvature $n(n-1)r$, $r > 1$ and the Riemannian product $\mathbb{S}^m(c) \times \mathbb{S}^{n-m}(\sqrt{1-c^2})$, $1 \leq m \leq n-2$ with constant scalar curvature $n(n-1)$ in $\mathbb{S}^{n+1}(1)$, $n \geq 3$.

**Example 3.1.** Let $M$ be a totally umbilical and non-totally geodesic hypersurface with constant scalar curvature $n(n-1)r$, $r > 1$ in $\mathbb{S}^{n+1}(1)$. In this case, $\Box = (n-1)H\Delta$, and from $S = nH^2$ and the Gauss equation (2.6) we get $H = \sqrt{r-1}$. By (1.2) we get

$$J_s = -\Box - \{n(n-1)H + nHS - f_3\} = -(n-1)\Delta + n(n-1)H(1 + H^2),$$

hence the eigenvalues $\lambda_i^J$ of $J_s$ are given by

$$\lambda_i^J = (n-1)H\lambda_i^\Delta - n(n-1)H(1 + H^2),$$

where $\lambda_i^\Delta$ denotes the eigenvalues of $\Delta$ (see Definition 1). It is well-known that $\lambda_1^\Delta = 0$, $\lambda_2^\Delta = nr = n(1 + H^2)$, hence we get

$$\lambda_1^J = -n(n-1)H(1 + H^2) = -n(n-1)r\sqrt{r-1} < 0,$$

$$\lambda_2^J = (n-1)H \cdot n(1 + H^2) - n(n-1)H(1 + H^2) = 0.$$  \hspace{1cm} (3.1)

**Example 3.2.** Let $M$ be the Riemannian product

$$\mathbb{S}^m(c) \times \mathbb{S}^{n-m}(\sqrt{1-c^2}), 1 \leq m \leq n-2, c = \sqrt{(n-1)m + \sqrt{(n-1)m(n-m)}}/n(n-1)$$

in $\mathbb{S}^{n+1}(1)$, $n \geq 3$. In this case, the position vector is

$$x = (x_1, x_2) \in \mathbb{S}^m(c) \times \mathbb{S}^{n-m}(\sqrt{1-c^2})$$

and the unit normal vector at this point $x$ is given by $e_{n+1} = (\sqrt{1-c^2}/c, -\sqrt{1-c^2}/c)$. Its principal curvatures are given by

$$k_1 = \cdots = k_m = -\sqrt{1-c^2}/c, k_{m+1} = \cdots = k_n = c/\sqrt{1-c^2}.$$  \hspace{1cm} (3.2)

Since the principal curvatures are constant hence $H$, $S$, $f_3$ are all contact given by

$$H = \frac{nc^2 - m}{cn\sqrt{1-c^2}},$$

$$S = \frac{m(1-c^2)}{c^2} + \frac{(n-m)c^2}{1-c^2} = n^2H^2,$$

$$f_3 = -\frac{m(1-c^2)^{3/2}}{c^3} + \frac{(n-m)c^3}{(1-c^2)^{3/2}}.$$  \hspace{1cm} (3.3)

After a long but straightforward computation, we know that $M$ has constant scalar curvature $n(n-1)$ and

$$H_3 = -\frac{2H}{n-2} = -\frac{2(nc^2 - m)}{cn(n-2)\sqrt{1-c^2}} < 0.$$  \hspace{1cm} (3.4)
hence the Jacobi operator $J_s$ is elliptic (cf. Proposition 1.5 in [11]). We also have
\[
n(n - 1)H + nHS - f_3 = \frac{(n - 2m)(n - 1)c^4 + 2m(m - 1)c^2 - m(m - 1)}{c^3(1 - c^2)^{3/2}},
\] (3.5)
and the Jacobi operator $J_s = -\Box - \{n(n - 1)H + nHS - f_3\}$ becomes
\[
J_s = -\Box - \frac{(n - 2m)(n - 1)c^4 + 2m(m - 1)c^2 - m(m - 1)}{c^3(1 - c^2)^{3/2}},
\] (3.6)
hence, the eigenvalues $\lambda^J_s$ of $J_s$ are given by
\[
\lambda^J_s = \lambda^\Box = -\frac{(n - 2m)(n - 1)c^4 + 2m(m - 1)c^2 - m(m - 1)}{c^3(1 - c^2)^{3/2}},
\] (3.7)
where $\lambda^\Box$ denotes the eigenvalues of the differential operator $\Box$ (see Definition 1).

Since the differential operator $\Box$ is self-adjoint and $M$ is compact, we have $\lambda^\Box_1 = 0$ and its corresponding eigenfunctions are non-zero constant functions, hence
\[
\lambda^J_s = -\frac{(n - 2m)(n - 1)c^4 + 2m(m - 1)c^2 - m(m - 1)}{c^3(1 - c^2)^{3/2}}.
\] (3.8)

Let $\{e_1, \cdots, e_n\}$ be a local orthonormal basis of $TM$ with dual basis $\{\omega_1, \cdots, \omega_n\}$ such that $\{e_1, \cdots, e_m\}$ is a local orthonormal basis of $TS^m(c)$ when restricted on $S^m(c)$, and $\{e_{m+1}, \cdots, e_n\}$ is a local orthonormal basis of $TS^{n-m}(\sqrt{1 - c^2})$ when restricted on $S^{n-m}(\sqrt{1 - c^2})$. So we have
\[
\Box f = \sum_{i=1}^{m}(nH - k_1)f_{ii} + \sum_{j=m+1}^{n}(nH - k_n)f_{jj} = (nH - k_1)\Delta_1 f + (nH - k_n)\Delta_2 f,
\] (3.9)
where $\Delta_1$ and $\Delta_2$ denote the Laplace-Beltrami operator on $S^m(c)$ and $S^{n-m}(\sqrt{1 - c^2})$ respectively. Hence, we conclude that
\[
\lambda^\Box_2 = \min \{(nH - k_1)\lambda^{\Delta_1}_2, (nH - k_n)\lambda^{\Delta_2}_2\},
\] (3.10)
where $\lambda^{\Delta_1}_2$ and $\lambda^{\Delta_2}_2$ are the second eigenvalues (or the first non-zero eigenvalue) of $\Delta_1$ and $\Delta_2$ which are given by
\[
\lambda^{\Delta_1}_2 = \frac{m}{c^2}, \lambda^{\Delta_2}_2 = \frac{n - m}{1 - c^2}.
\] (3.11)
Therefore, from (3.7), (3.10) and (3.11), after a direct computation, we have
\[
\lambda^{J_s}_2 = \min\{(nH - k_1)\frac{m}{c^2} - \frac{(n - 2m)(n - 1)c^4 + 2m(m - 1)c^2 - m(m - 1)}{c^3(1 - c^2)^{3/2}},
\]
\[
\frac{(nH - k_n)\frac{n - m}{1 - c^2} - \frac{(n - 2m)(n - 1)c^4 + 2m(m - 1)c^2 - m(m - 1)}{c^3(1 - c^2)^{3/2}}}{c^3(1 - c^2)^{3/2}}
\] (3.12)
\[
\min\left\{\frac{(n - m)[(1 - n)c^2 + mc^2]}{c^3(1 - c^2)^{3/2}}, \frac{m[(n - 1)c^2 - (m - 1)]}{c^3(1 - c^2)^{3/2}}\right\}.
\]
Since $c = \sqrt{\frac{(n-1)m+(n-1)m(n-m)}{n(n-1)}}$, we have
\[
\frac{(n - m)[(1 - n)c^4 + mc^2]}{c^3(1 - c^2)^{3/2}} - \frac{m[(n - 1)c^2 - (m - 1)]}{c^3(1 - c^2)^{3/2}}
\] (3.13)
\[
= \frac{-n(n - 1)c^4 + 2m(1 - n)c^2 + m(m - 1)}{c^3(1 - c^2)^{3/2}} = 0,
\]
By using (3.8) and (3.12), we get
\[
\lambda_1^J = -\frac{(n - 2m)(n - 1)c^4 + 2m(m - 1)c^2 - m(m - 1)}{c^3(1 - c^2)^{3/2}},
\]
(3.14)
and
\[
\lambda_2^J = \min \left\{ \frac{(n - m)((1 - n)c^4 + mc^2)}{c^3(1 - c^2)^{3/2}}, \frac{m(c^2 - 1)((n - 1)c^2 - (m - 1))}{c^3(1 - c^2)^{3/2}} \right\}
\]
\[
= \frac{(n - m)((1 - n)c^2 + m)}{c(1 - c^2)^{3/2}} < 0.
\]
(3.15)
On the other hand, we also have
\[
-\frac{(n - 2m)(n - 1)c^4 + 2m(m - 1)c^2 - m(m - 1)}{c^3(1 - c^2)^{3/2}} + 2n(n - 1)H
\]
\[
= -\frac{(2c^2 - 1)(n(n - 1)c^4 + 2m(1 - n)c^2 + m(m - 1))}{c^3(1 - c^2)^{3/2}} = 0,
\]
(3.16)
\[
\frac{(n - m)((1 - n)c^2 + m)}{c(1 - c^2)^{3/2}} + n(n - 1)H
\]
\[
= -\frac{n(n - 1)c^4 + 2m(1 - n)c^2 + m(m - 1)}{c(1 - c^2)^{3/2}} = 0,
\]
(3.17)
and
\[
\frac{(n - m)((1 - n)c^2 + m)}{c(1 - c^2)^{3/2}} - \frac{n(n - 1)(n - 2)}{2}H_3
\]
\[
= -\frac{(n(n - 1)c^4 + 2m(1 - n)c^2 + m(m - 1))(c^2(2n - 1) - 2m + 1)}{c^3(1 - c^2)^{3/2}} = 0,
\]
(3.18)
so we get
\[
\lambda_1^J = -2n(n - 1)H < \lambda_2^J = -n(n - 1)H = \frac{n(n - 1)(n - 2)}{2}H_3.
\]
(3.19)
In the following we will assume that \( x: M \to \mathbb{S}^{n+1}(1) \) is an \( n \)-dimensional compact orientable hypersurface with constant scalar curvature \( n(n - 1)r, \ r \geq 1, \) in \( \mathbb{S}^{n+1}(1), \ n \geq 5, \) when \( r = 1, \) we assume moreover that \( H_3 \neq 0. \) We can choose a unit normal vector \( e_{n+1} \) such that \( H > 0. \)

Let \( a \) be a fixed vector in \( \mathbb{R}^{n+2}. \) We define functions \( f^a: M \to \mathbb{R} \) and \( \tilde{g}^a: M \to \mathbb{R} \) by
\[
f^a = \langle a, x \rangle, \ \tilde{g}^a = \langle a, e_{n+1} \rangle.
\]
(3.20)
By using the structure equations and the definition of the covariant derivatives, we have the following result.

Lemma 3.3. (see [4]) The gradient and the second derivative of the functions \( f \) and \( \tilde{g} \) are given by
\[
f_i^a = \langle a, e_i \rangle, \ f_{ij}^a = \tilde{g}^a h_{ij} - f^a \delta_{ij},
\]
\[
\tilde{g}_j^a = -\sum_{i=1}^n \langle a, e_i \rangle h_{ij}, \ \tilde{g}^a_{jk} = -\sum_{i=1}^n \langle a, e_i \rangle h_{ijk} - \sum_{i=1}^n \tilde{g}^a h_{ij} h_{ik} + f^a h_{jk}.
\]
(3.21)
Proof. By (2.1) we get
\[ df^a = < a, dx > = \sum_i < a, e_i > \omega_i, \]
hence from (2.8) we get
\[ f_i^a = < a, e_i >. \]  
(3.22)

From (2.2) and (3.22) we get
\[ \sum_{j=1}^n f_{ij}^a \omega_j = df_i + \sum_{j=1}^n f_{j} \omega_{ji} = < a, de_i > + \sum_{j=1}^n < a, e_j > \omega_{ji} = \sum_{j=1}^n < a, e_{n+1} > h_{ij} \omega_j - < a, x > \omega_i, \]
hence we get
\[ f_{ij}^a = < a, e_{n+1} > h_{ij} - < a, x > \delta_{ij} = \tilde{g}_{i}^a h_{ij} - f^a \delta_{ij}, \]  
(3.23)

After an analogous argument, we can get
\[ \tilde{g}_j^a = - \sum_{i=1}^n < a, e_i > h_{ij}, \quad \tilde{g}_{jk}^a = - \sum_{i=1}^n < a, e_i > h_{ijk} - \sum_{i=1}^n \tilde{g}_i^a h_{ij} h_{ik} + f^a h_{jk}. \]  
(3.24)

We will use a technique which was introduced by Li and Yau in [13] and was later used by other authors (see [14], [16] and [21]).

Let \( B^{n+2} \) be the open unit ball in \( \mathbb{R}^{n+2} \). For each point \( g \in B^{n+2} \) we consider the map
\[ F_g(p) = \frac{p + (\mu < p, g > + \lambda)g}{\lambda(< p, g > + 1)}, \quad \forall \ p \in \mathbb{S}^{n+1}(1) \subset \mathbb{R}^{n+2}, \]  
(3.25)
where \( \lambda = (1 - \|g\|^2)^{-1/2}, \mu = (\lambda - 1)\|g\|^2 \) and \(<,>\) denotes the usual inner product on \( \mathbb{R}^{n+2} \).

A direct computation (see [14], [21]) shows that \( F_g \) is a conformal transformation from \( \mathbb{S}^{n+1}(1) \) to \( \mathbb{S}^{n+1}(1) \) and the differential map \( dF_g \) of \( F_g \) is given by
\[ dF_g(v) = \lambda^{-2}(< p, g > + 1)^{-2}\{\lambda(< p, g > + 1)v - \lambda < v, g > p + < v, g > (1 - \lambda)\|g\|^2 g\}, \]
where \( v \) is a tangent vector to \( \mathbb{S}^{n+1} \) at the point \( p \). Hence, for two vectors \( v, w \in T_p \mathbb{S}^{n+1} \) we have (see [16], [21])
\[ < dF_g(v), dF_g(w) > = \frac{1 - \|g\|^2}{(< p, g > + 1)^2} < v, w >. \]

By use of the technique in Li-Yau [13], we have the following result:

**Lemma 3.4.** (see [17], [16] and [21])

Let \( x : M \to \mathbb{S}^{n+1} \) be a compact hypersurface in \( \mathbb{S}^{n+1} \) with constant scalar curvature \( n(n - 1)r, \ r \geq 1, \) and \( u \) be a first positive eigenfunction of the Jacobi operator \( J_s \) on \( M, \) then there exists \( g \in B^{n+2} \) such that \( \int_M u(F_g \circ x)dv = (0, \ldots, 0). \)

Let \( \{E^A\}_{A=1}^{n+2} \) be a fixed orthonormal basis of \( \mathbb{R}^{n+2} \), for a fixed point \( g \in B^{n+2} \), we define functions \( f^A : M \to \mathbb{R} \) by
\[ f^A = < E^A, F_g \circ x > = \frac{< E^A, x > + (\mu < x, g > + \lambda) < g, E^A >}{\lambda(< x, g > + 1)}, \]  
(3.26)
Lemma 3.5. The gradient of $f^A$ are given as by

$$f^A_i = \frac{<E^A, e_i>}{\lambda(<x, g > + 1)} + \frac{<g, e_i>}{\lambda(<x, g > + 1)^2}(-<E^A, x> + \frac{1 - \lambda}{\lambda\|g\|^2} <g, E^A>).$$ \hspace{1cm} (3.27)

Proof. By applying Lemma 3.3, we have

$$f^A_i = \frac{<E^A, e_i> + \mu <g, e_i> + <g, E^A>}{\lambda(<x, g > + 1)} - \frac{(<E^A, x> + (\mu <x, g > + \lambda) <g, E^A>) <g, e_i>}{\lambda(<x, g > + 1)^2}$$

$$= \frac{<E^A, e_i>}{\lambda(<x, g > + 1)} + \frac{<g, e_i>}{\lambda(<x, g > + 1)^2}(-<E^A, x> - \lambda <g, E^A>)$$

$$= \frac{<E^A, e_i>}{\lambda(<x, g > + 1)} + \frac{<g, e_i>}{\lambda(<x, g > + 1)^2}(-<E^A, x> + \frac{1 - \lambda}{\lambda\|g\|^2} <g, E^A>).$$

\[\square\]

We also need the following Lemma 3.6, Lemma 3.7 and Lemma 3.8 to estimate the second
eigenvalue $\lambda^2_2$ of the Jacobi operator $J_s$ on $M$.

Lemma 3.6. Let $M$ be an $n$-dimensional compact hypersurface with constant scalar curvature
$n(n - 1)r$, $r \geq 1$, in $S^{n+1}(1)$. Let $f^A$ be the functions given by (3.26), we have

$$\sum_{A=1}^{n+2} \int_M (J_s f^A) dv = \int_M \frac{n(n - 1)H(1 - \|g\|^2)}{2} dv - \int_M \{\frac{n(n - 1)}{2} (2H - (n - 2)H_3 + nHH_2)\} dv. \hspace{1cm} (3.28)$$

Proof. By divergence theorem and Lemma 3.5 we have

$$-\sum_{A=1}^{n+2} \int_M (\square f^A : f^A) dv = \sum_{A=1}^{n+2} \int_M (nH\delta_{ij} - h_{ij}) f^A_i f^A_j dv$$

$$= \sum_{A=1}^{n+2} \int_M (nH\delta_{ij} - h_{ij})(\frac{<E^A, e_i>}{\lambda(<x, g > + 1)} + \frac{<g, e_i>}{\lambda(<x, g > + 1)^2}(-<E^A, x> + \frac{1 - \lambda}{\lambda\|g\|^2} <g, E^A>))$$

$$\cdot (\frac{<E^A, e_j>}{\lambda(<x, g > + 1)} + \frac{<g, e_j>}{\lambda(<x, g > + 1)^2}(-<E^A, x> + \frac{1 - \lambda}{\lambda\|g\|^2} <g, E^A>)) dv$$

$$= \sum_{A=1}^{n+2} \int_M \{ \int_M (nH\delta_{ij} - h_{ij})(\frac{\delta_{ij}}{\lambda^2(<x, g > + 1)^3} + \frac{<g, e_i> <g, e_j>}{\lambda^2(<x, g > + 1)^4}(-<E^A, x> + \frac{1 - \lambda}{\lambda\|g\|^2} <g, E^A>)^2$$

$$+ \frac{<E^A, e_i> <g, e_j> + <E^A, e_j> <g, e_i>}{\lambda^2(<x, g > + 1)^3}(-<E^A, x> + \frac{1 - \lambda}{\lambda\|g\|^2} <g, E^A>)) dv$$

$$= \int_M (nH\delta_{ij} - h_{ij})(\frac{\delta_{ij}}{\lambda^2(<x, g > + 1)^2}) dv$$

$$= \int_M \frac{n(n - 1)H(1 - \|g\|^2)}{2} dv,$$

where we use the fact that $\sum_{A=1}^{n+2} <E_A, e_i> <E_A, e_j> = <e_i, e_j> = \delta_{ij}$ in the third equality.
Lemma 3.7. Let \( H \) and the equality holds if and only if \( S = n^2H^2 - n(n-1)H_2 \).

By Newton formula, we get
\[
\begin{align*}
J &= n^3H^3 + \frac{n(n-1)(n-2)}{2}H_3 - \frac{3n^2(n-1)}{2}HH_2, \\
S &= n^2H^2 - n(n-1)H_2.
\end{align*}
\]

Hence, \( J_s \) becomes
\[
\begin{align*}
J_s &= -\Delta - \{n(n-1)H + nH(n^2H^2 - n(n-1)H_2) - (n^3H^3 + \frac{n(n-1)(n-2)}{2}H_3 - \frac{3n^2(n-1)}{2}HH_2)\} \\
&= -\Delta - n(n-1)H - \frac{n^2(n-1)}{2}HH_2 + \frac{n(n-1)(n-2)}{2}H_3 \\
&= -\Delta - \frac{n(n-1)}{2}(2H - (n-2)H_3 + nHH_2).
\end{align*}
\]

Then by using the fact that
\[
\sum_{A=1}^{n+2} f_A^2 \cdot f_A^2 = \sum_{A=1}^{n+2} < E^A, x > \cdot < E^A, x > = < x, x > = 1,
\]
we can immediately get \( (3.28) \).

\( \Box \)

For a fixed point \( g \in B^{n+2} \), let
\[
f = < x, g >, \quad \tilde{g} = < e_{n+1}, g >, \quad \rho = \ln \lambda - \ln (1 + f),
\]
where \( \lambda = (1 - \|g\|^2)^{-1/2} \), we get
\[
e^{2\rho} = \frac{1 - \|g\|^2}{(1 + f)^2} = \frac{1 - \|g\|^2}{(x, g > +1)^2}, \quad \rho_i = \frac{-f_i}{1 + f}, \quad \rho_{ij} = \frac{-f_if_j}{1 + f} + \frac{f_if_j}{(1 + f)^2}.
\]

Lemma 3.7. Let \( x : M \to S^{n+1}(1) \) be an \( n \)-dimensional compact hypersurface with constant scalar curvature \( n(n-1)r \), \( r \geq 1 \), in \( S^{n+1}(1) \). When \( r = 1 \), we assume moreover that \( H_3 \neq 0 \).

We can choose a unit normal vector \( e_{n+1} \) such that \( H > 0 \). Let \( \rho \) be the function defined by \( (3.33) \), then we have
\[
\int_M \frac{H(1 - \|g\|^2)}{(x, g > +1)^2}dv \leq \int_M (H + \frac{H_2}{H})dv - \int_M [H\|\nabla\rho\|^2 - \frac{2}{n(n-1)}(nH\delta_{ij} - h_{ij})\rho_i\rho_j]dv,
\]
and the equality holds if and only if \( H_2 + \frac{\delta H}{1+f} \equiv 0 \) on \( M \).

Proof. By definition of \( \rho \) we have \( e^{2\rho} = \frac{1 - \|g\|^2}{(x, g > +1)^2} \).

We have
\[
(nH\delta_{ij} - h_{ij})\rho_i\rho_j = (nH\delta_{ij} - h_{ij})f_if_j/(1 + f)^2 = \frac{nH\|\nabla f\|^2}{(1 + f)^2} - \frac{h_{ij}f_if_j}{(1 + f)^2}.
\]

and
\[
\Box \rho = (nH\delta_{ij} - h_{ij})\rho_{ij}
\]
\[
= (nH\delta_{ij} - h_{ij})(-\frac{f_if_j}{1 + f} + \frac{f_if_j}{(1 + f)^2})
\]
\[
= -\Delta f - \frac{nH\|\nabla f\|^2}{1 + f} + \frac{h_{ij}f_if_j}{1 + f} - \frac{h_{ij}f_if_j}{(1 + f)^2}.
\]
From (3.34), (3.36) and (3.37) and by using Lemma 3.3, we get
\[
(\rho - (nH\delta_{ij} - h_{ij})\rho_i\rho_j) \cdot \frac{2}{n(n-1)} + e^{2\rho}H
\]
\[
= \left( -\Delta f + h_{ij}f_{ij} \right) \cdot \frac{2}{n(n-1)} + H(1 - \|g\|^2)\]
\[
= \left( -nH(nH\tilde{g} - nf) + h_{ij}(\tilde{g}h_{ij} - f\delta_{ij}) \right) \cdot \frac{2}{n(n-1)} + H(1 - \|g\|^2)
\]
\[
= \frac{2Hf - 2H_2\tilde{g}}{1 + f} + H(1 - f^2 - f\tilde{g}^2 - \tilde{g})^2 \quad \text{(3.38)}
\]
\[
= H - \frac{Hf^2}{(1 + f)^2} - H\|\nabla\rho\|^2 - \frac{(H_2 + \tilde{g}Hf)\tilde{g}}{H} + \frac{H_2}{H}
\]
which immediately implies
\[
\int_M \frac{H(1 - \|g\|^2)}{\|x, g\| + 1}^2 \, dv = \int_M e^{2\rho}H \, dv
\]
\[
= \int_M \left[ H + \frac{H_2}{H} - H\|\nabla\rho\|^2 + \frac{2}{n(n-1)}(nH\delta_{ij} - h_{ij})\rho_i\rho_j - \frac{(H_2 + \tilde{g}Hf)\tilde{g}}{H} \right] \, dv.
\]

Hence we can immediately get the inequality (3.35) and the equality holds if and only if \( H_2 + \tilde{g}Hf \equiv 0 \) on \( M \).

**Lemma 3.8.** Let \( M \) be an \( n \)-dimensional compact hypersurface with constant scalar curvature \( n(n-1)r \), \( r \geq 1 \), in \( S^{n+1} \), \( n \geq 5 \). When \( r = 1 \), we assume moreover that \( H_3 \neq 0 \). We choose a unit normal vector \( e_{n+1} \) such that \( H > 0 \), then we have
\[
\int_M \left[ H\|\nabla\rho\|^2 - \frac{2}{n(n-1)}(nH\delta_{ij} - h_{ij})\rho_i\rho_j \right] \, dv \geq 0.
\]

**Proof.** \( \forall \rho \in M \), let \( k_i \) denote the principal curvatures of \( M \) at \( p \), we choose a orthonormal basis such that \( h_{ij} = \delta_{ij}k_i \). By Gauss equation (2.3), we have
\[
n^2H^2 - \sum_i k_i^2 = n(n-1)(r - 1) \geq 0
\]
Since we can choose a unit normal vector \( e_{n+1} \) such that \( H > 0 \), we get from (3.40)
\[
nH \geq |k_i|, \ \forall 1 \leq i \leq n.
\]
As \( n \geq 5 \), we have \( \frac{n(n-3)}{2}H \geq nH \), so we have
\[
H\|\nabla\rho\|^2 - \frac{2}{n(n-1)}(nH\delta_{ij} - h_{ij})\rho_i\rho_j
\]
\[
= H\sum_i \rho_i^2 - \frac{2}{n(n-1)}(nH\delta_{ij} - \delta_{ij}k_i)\rho_i\rho_j
\]
\[
= H\sum_i \rho_i^2 - \sum_i \frac{2}{n(n-1)}(nH - k_i)\rho_i^2
\]
\[
= \frac{2}{n(n-1)} \sum_i \rho_i^2\left( \frac{n(n-3)}{2}H + k_i \right) \geq \frac{2}{n(n-1)} \sum_i \rho_i^2(nH - |k_i|) \geq 0.
\]
Hence, we get (3.39).
4 Proofs of Theorem 1.3 and Theorem 1.4

Proof of Theorem 1.3: Since \( r > 1 \), we have \( \Box \) is an elliptic operator and \( H \neq 0 \), we can choose a unit normal vector \( e_{n+1} \) such that \( H > 0 \). Let \( u \) be a first eigenfunction of \( J_s \), we can assume that \( u \) is positive on \( M \), by Lemma 3.4 there exists \( g \in B^{n+2} \) such that

\[
\int_M u(F_g \circ x)dv = (0, \ldots, 0), \tag{4.1}
\]

which implies that the functions \( f^A \) given by (3.26) are perpendicular to the function \( u \), i.e., \( \int_M u \cdot f^Adv = 0 \). Then by using the min-max characterization of eigenvalues for elliptic operators, we have

\[
\lambda_{2s} \cdot \int_M (f^A \cdot f^A)dv \leq \int_M (J_s f^A \cdot f^A)dv, \quad \forall 1 \leq A \leq n + 2. \tag{4.2}
\]

Summing up and using the fact that \( \sum_{A=1}^{n+2} f^A \cdot f^A = 1 \) (see (3.32)), we obtain

\[
\lambda_{2s} \cdot Vol(M) \leq \sum_{A=1}^{n+2} \int_M (J_s f^A \cdot f^A)dv. \tag{4.3}
\]

Combining with Lemma 3.6 and (4.3) we get

\[
\lambda_{2s} \cdot Vol(M) \leq \int_M \frac{n(n-1)H}{n} \frac{1 - ||g||^2}{(x, g > +1)^2} dv - \int_M \frac{n(n-1)}{2} (2H - (n - 2)H_3 + nHH_2)dv. \tag{4.4}
\]

Then by (4.4), Lemma 3.7 and Lemma 3.8, we get

\[
\lambda_{2s} \cdot Vol(M) \leq n(n-1) \int_M (H + \frac{H^2}{2})dv - \int_M \frac{n(n-1)}{2} (2H - (n - 2)H_3 + nHH_2)dv
\]

\[
= n(n-1) \int_M \left( \frac{H^2}{2} + \frac{n-2}{2} H_3 - \frac{nHH_2}{2} \right)dv. \tag{4.5}
\]

From definition of \( H_2 \) and the Gauss equation (2.6) we have

\[
H_2 = r - 1 = \text{constant} > 0. \tag{4.6}
\]

So we get \( H_3 \leq \frac{H_2^2}{HH} \) and \( H_2 \leq H^2 \) (see [10], p. 52) and hence

\[
\lambda_{2s} \cdot Vol(M) \leq n(n-1) \int_M \left( \frac{H^2}{2} + \frac{n-2}{2} H_3 - \frac{nHH_2}{2} \right)dv
\]

\[
\leq n(n-1) \int_M \left( \frac{H^2}{2} + \frac{n-2}{2} H_3 - \frac{nHH_2}{2} \right)dv \tag{4.7}
\]

\[
= n(n-1) \int_M \frac{nH_2}{2} (\frac{H_2}{H} - H)dv \leq 0,
\]

therefore we get \( \lambda_{2s} \leq 0 \).

When \( \lambda_{2s} = 0 \), then all the inequalities become equalities. From (4.7) we have \( H_2 = H^2 \) on \( M \), since \( H_2 \) is a positive constant, we get \( M \) is a totally umbilical and non-totally geodesic hypersurface with constant scalar curvature \( n(n-1)r \). On the other hand, if \( M \) is a totally umbilical and non-totally geodesic hypersurface with constant scalar curvature \( n(n-1)r \), from Example 3.1 in section 3, we know that \( \lambda_{2s} = 0 \). \( \Box \)
Remark 4.1. We notice that from (4.7) we can get a more precise upper bound for $\lambda_{2}^{J_{s}}$, that is,

\[
\lambda_{2}^{J_{s}} \leq n(n - 1)(\frac{H_{2}^{2}}{\min H} + \frac{n - 2}{2} \max H_{3} - \frac{nH_{2}}{2} \min H) = n(n - 1)(\frac{(r - 1)^{2}}{\min H} + \frac{n - 2}{2} \max H_{3} - \frac{n(r - 1)}{2} \min H).
\]

(4.8)

Proof of Theorem 1.4: Since $r = 1$, from (4.6) we have $H_{2} = 0$. Since we assume that $H_{3}$ does not vanish on $M$, we get $J_{s}$ is elliptic and the mean curvature $H$ does not vanish on $M$ (cf. Proposition 1.5 in [1]). If we choose a unit normal vector $e_{n+1}$ such that $H > 0$, then $H_{3} \leq \frac{H_{2}^{2}}{H} = 0$. Since we assume that $H_{3} \neq 0$ on $M$, we get $H_{3} < 0$. As Lemma 3.6, Lemma 3.7 and Lemma 3.8 hold for both the case $r > 1$ and the case $r = 1$, after a analogous argument with the proof of Theorem 1.3, we get (4.1)-(4.5) still hold in this case, hence we get

\[
\lambda_{2}^{J_{s}} \cdot \text{Vol}(M) \leq n(n - 1) \cdot \left[\int_{M} (\frac{H_{2}^{2}}{H} + \frac{n - 2}{2} H_{3} - \frac{nH_{2}}{2} \min H) dv \right] = \frac{n(n - 1)(n - 2)}{2} \cdot \left[\int_{M} H_{3} dv \right] \leq \frac{n(n - 1)(n - 2)}{2} \max H_{3} \cdot \text{Vol}(M) = -\frac{n(n - 1)(n - 2)}{2} \min |H_{3}| \cdot \text{Vol}(M).
\]

Hence, we get

\[
\lambda_{2}^{J_{s}} \leq -\frac{n(n - 1)(n - 2)}{2} \min |H_{3}|.
\]

(4.9)

(4.10)

When $\lambda_{2}^{J_{s}} = -\frac{n(n - 1)(n - 2)}{2} \min |H_{3}|$, then the inequalities in (4.35), (4.2) and (4.9) become equalities. The equality in (4.9) holds implies that $H_{3} = \text{constant} \neq 0$. Since $H_{2} = 0$, the equalities in (4.35) holds implies that $g = < g, e_{n+1} > \equiv 0$ on $M$. We claim that $g$ must be 0, otherwise, we have that $M$ is a hypersphere (cf. [15]), hence $M$ is totally umbilical, since $H_{2} = 0$, we immediately get $M$ is totally geodesic which is a contradiction with $H_{3} \neq 0$. Hence we have $g \equiv 0$, from (3.20) we get $f^{A} = < E^{A}, F_{3} \circ x >= < E^{A}, x >$, which means $\{f^{A}, 1 \leq A \leq n + 2\}$ are the position functions of $x : M \to S^{n+1}(1)$. Since the equality in (4.2) holds, we get that the position functions $\{f^{A} = < E^{A}, x >, 1 \leq A \leq n + 2\}$ must be the second eigenfunctions of $J_{s}$ corresponding to $\lambda_{2}^{J_{s}}$.

On the other hand, if we assume that $H_{3} = \text{constant} \neq 0$ and the position functions $\{f^{A} = < E^{A}, x >, 1 \leq A \leq n + 2\}$ are the second eigenfunctions of $J_{s}$ corresponding to $\lambda_{2}^{J_{s}}$. We can choose a unit normal vector $e_{n+1}$ such that $H > 0$, $H_{3} < 0$.

Since $H_{2} = 0$, by using (1.1) and (3.21), we get

\[
\Box \tilde{f}^{A} = n(n - 1)H_{2} < a, e_{n+1} > -n(n - 1)H \tilde{f}^{A} = -n(n - 1)H \tilde{f}^{A}, \forall 1 \leq A \leq n + 2,
\]

then from (1.2) and (3.30) we have

\[
J_{s}\tilde{f}^{A} = n(n - 1)H \tilde{f}^{A} - \{n(n - 1)H + nHS - f_{3}\} \tilde{f}^{A} = (f_{3} - nHS) \tilde{f}^{A} = \{(n^{3}H^{3} + \frac{n(n - 1)(n - 2)}{2} H_{3} - \frac{3n^{2}(n - 1)}{2} H_{2}H_{3}) - (n^{3}H^{3} - n^{2}(n - 1)H_{2})\} \tilde{f}^{A} = \frac{n(n - 1)(n - 2)}{2} H_{3} \tilde{f}^{A}, \forall 1 \leq A \leq n + 2,
\]

(4.8)
hence we get $\lambda_2^{J_s} = \frac{n(n-1)(n-2)}{2} H_3 = -\frac{n(n-1)(n-2)}{2} \min |H_3|$. In particular, when $M$ is the Riemannian product $S^m(c) \times S^{n-m}\sqrt{1-c^2}$, $1 \leq m \leq n-2$, $c = \sqrt{\frac{(n-1)m+\sqrt{(n-1)m(n-m)}}{n(n-1)}}$ in $S^{n+1}(1)$, from Example 3.3 in section 3, we know that in this case the equality in (4.10) is attained.

Remark 4.2. Since Lemma 3.10 does not hold when $n = 3$ and $n = 4$, we can not prove Theorem 1.3 and Theorem 1.4 by our technique in $n = 3$ and $n = 4$. So it is an interesting problem to get the estimate for the second eigenvalue of the Jacobi operator $J_s$ of the hypersurface $x : M^n \to S^{n+1}(1)$ when $n = 3$ and $n = 4$. 

\[ \]
References


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