Conference on

Interplay between representation theory and geometry

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Dedicated to the 65th birthday of
Professor Claus Michael Ringel and Professor Kyoji Saito

Programme and Abstract

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Modular branching rule for the affine Hecke algebra of type A

Susumu Ariki (Osaka University)

By work of Ginzburg and Lusztig, we know that irreducible modules over the extended affine Hecke algebra of type A defined over the field of complex numbers are parametrized by aperiodic multisegments. The construction of irreducible modules is by geometric methods, and it is difficult to know explicit representing matrices. On the other hand, we know how to construct irreducible modules by a purely algebraic method. Hence, it is important to know the module correspondence between the two constructions. Our result is that a crystal embedding gives the module correspondence. It is a corollary of the modular branching rule proved by the speaker, Jacon and Lecouvey.

The F-inflation category and its stable category

Xiao-Wu Chen (University of Paderborn)

Let F be an exact functor between two Frobenius categories such that it sends projective objects to projective objects. We consider the F-inflation category, which turns out to be a Frobenius category. By a general result of Happel, its stable category is triangulated. We will show that such a construction gives rise naturally to a recollement of triangulated categories; moreover, this construction is compatible with tilting objects. The construction is related to work by Iyama-Kato-Miyachi, Kussin-Lenzing-Meltzer, Li-Zhang and Ringel-Schmidmeier. This is based on a project with Sefi Ladkani.

Linked Fuchsian differential equations and multiplicative preprojective algebras

William Crawley-Boevey (University of Leeds)

Multiplicative preprojective algebras associated to quivers were introduced by Crawley-Boevey and Shaw in order to study (in case the underlying quiver is star-shaped) problems concerning the monodromy of differential equations on the Riemann sphere. I will show how multiplicative preprojective algebras for more general quivers arise when considering collections of Fuchsian differential equations with linked residues, and I will discuss an application to the structure of multiplicative preprojective algebras.

Piecewise hereditary algebras

Dieter Happel (Chemnitz University of Technology)

Let Λ be a finite dimensional algebra over an algebraically closed field k and let $\operatorname{mod}\Lambda$ be the category of finite dimensional left Λ-modules. Recall that Λ is said to be piecewise hereditary if there exists a hereditary abelian category $\mathcal{H}$ such that the bounded derived categories $D^b(\operatorname{mod}\Lambda)$ and $D^b(\mathcal{H})$ are equivalent as triangulated categories.

We will give various equivalent conditions for an algebra to be piecewise hereditary and state some useful homological properties.

Amongst those we state the homological characterization obtained in joint work with Dan Zacharia in terms of the strong global dimension.

Moreover we will report on some joint work with Uwe Seidel on piecewise hereditary Nakayama algebras.

Also we will report on some recent joint work with Luise Unger on the representation dimension of piecewise hereditary algebras.

D-Branes and Categories

Kentaro Hori (University of Tokyo)

2-Auslander algebras associated with reduced words in Coxeter groups

Osamu Iyama (Nagoya University)

We investigate the endomorphism algebras of standard cluster tilting objects in the stably 2-Calabi-Yau categories $\mathrm{Sub}\Lambda_w$ with elements w in Coxeter groups introduced by Buan-Iyama-Reiten-Scott. They are examples of the 2-Auslander algebras, which are 3-dimensional analogue of Auslander algebras. Generalizing work by Geiss-Leclerc-Schroer we show that they are quasihereditary. We also describe the cluster tilting object giving rise to the Ringel dual, and prove that there is a duality between $\mathrm{Sub}\Lambda_w$ and the category $\mathcal{F}(\Delta)$ of good modules over the quasi-hereditary algebra. For a reduced word $w = uv$, we show the relationship between the 2-Calabi-Yau triangulated categories $\underline{\mathrm{Sub}}\Lambda_v$ and $\underline{\mathrm{Sub}}\Lambda_w$. As an application we show that a standard cluster tilting object M and its syzygy ΩM lie in the same component in the cluster tilting graph. This is a joint work with I. Reiten.

On A_∞-enhancements of triangulated categories and finite dimensional algebras

Hiroshige Kajiura (Chiba University)

For triangulated categories having DG or A_∞-enhancements, we discuss the relation between the A_∞-products and the triangulated structures. In particular, we explain that A_∞-enhancements of a triangulated category are not unique up to A_∞-homotopy equivalence in general, though there exist many triangulated categories whose A_∞-enhancements are unique. We also discuss how A_∞-categories appear in representation theory of finite dimensional algebras.

Standard objects and exceptional objects, and a box full of structure

Steffen König (University of Cologne)

The category of objects filtered by Verma or Weyl modules in Lie theory or by elements of an exceptional sequence in geometry carries interesting structures. Relations between these structures will be discussed, and applied to highest weight categories. (Report on a joint project with Sergiy Ovsiyenko)

Cosupport and colocalizing subcategories of modules and complexes

Henning Krause (Bielefeld University)

Recent work of Neeman shows how colocalizing subcategories of the derived category of a commutative noetherian ring can be classified. Colocalizing subcategories are triangulated subcategories that are closed under taking arbitrary products. The classification is remarkably simple and beautiful, despite all set theoretic complications one might expect. I will explain Neeman's result and will discuss consequences for the classification of representations of finite dimensional algebras. This leads to the notion of cosupport; it is based on joint work with Dave Benson and Srikanth Iyengar.

Flags for nilpotent operators and weighted projective lines

Helmut Lenzing (University of Paderborn)

My talk is on joint work with Dirk Kussin and Hagen Meltzer. We establish a link between singularity theory and the study of invariant flags for (graded) nilpotent linear operators. The link is provided through categories of vector bundles on weighted projective lines of weight type $(2, a, b)$, where a relates to the flag length and b gets an interpretation as the nilpotency degree. This relates to recent work by Kajiura-Saito-Takahashi (2007-2009), Ringel-Schmidmeier (2006-2008), Lenzing-de la Pena (2006-2008) and Simson (2002-2007).

Coxeter transformations determined by intersection matrices

Liangang Peng (Sichuan University)

Associated to every intersection matrix, one can define the Coxeter transformation. We mainly consider the order. In particular, we obtain that the order of the Coxeter transformation determined by a d-fold affinization matrix of a Cartan matrix keeps unchange with the order of the Coxeter transformation determined by the Cartan matrix. We also consider the Coxeter transformations under braid transformations.

Tilting theory and c-sortable words

Idun Reiten (Norwegian University of Science and Technology)

Let Q be a finite quiver without oriented cycles and C the associated Coxeter group. From work with Buan, Iyama and Scott (independently by Geiss, Leclerc, Schröer in the adaptable case) there is a finite dimensional algebra A naturally associated with a (reduced) word in C. For any reduced expression of w there is a cluster tilting object T in the stably 2-Calabi-Yau category $\operatorname{Sub} A$. There is a set of length(w) indecomposable A-modules naturally associated with w, called layers, where the indecomposable projective A-modules are filtered by the layers. For a class of words called c-sortable we discuss a relationship between these layers and tilting theory for $\operatorname{mod} kQ$. The lecture is based upon joint work with Amiot, Iyama and Todorov.

The set of lengths of indecomposables has no gaps

Claus Michael Ringel (Bielefeld University)

Let A be a finite-dimensional k-algebra with k algebraically closed. Bongartz has recently shown that the existence of an indecomposable A-module of length $n > 1$ implies that also indecomposable A-modules of length $n - 1$ exist. Using a slight modification of his arguments, we strengthen the assertion as follows: If there is an indecomposable module of length n, then there is also an accessible one.

Here, the accessible modules are defined inductively, as follows: First, the simple modules are accessible. Second, a module of length $n > 1$ is accessible provided it is indecomposable and there is a submodule or a factor module of length $n - 1$ which is accessible.

Spectral decomposition of the Coxeter elements of type $A_{\frac{1}{2}\infty}$ and $D_{\frac{1}{2}\infty}$

Kyoji Saito (University of Tokyo)

In order to understand the KP- and KdV-hierarchy (due to Witten and Kontsevich), we introduce two transcendental functions $f_A(x, y)$ and f_D of two variables. The vanishing cycles of them form infinite quivers of type $A_{\frac{1}{2}\infty}$ and $D_{\frac{1}{2}\infty}$, respectively. We study the Coxeter elements associated with them, and describe the spectra (or, exponents) of them. The spectra is no-more discrete but continuous on the interval $(0, 1)$.

On tensor category arising from representation theory of the restricted quantum universal enveloping algebra associated to $\mathfrak{sl}_2$

Yoshihisa Saito (University of Tokyo)

In this talk, we study the tensor structure of the module category of the restricted quantum enveloping algebra associated to $\mathfrak{sl}_2$. Indecomposable decomposition of all tensor products of modules over this algebra is completely determined in explicit formulas. As a by-product, we show that the module category of the restricted quantum enveloping algebra associated to $\mathfrak{sl}_2$ is not a braided tensor category.

Generic bases for cluster algebras

Jan Schröer (University of Bonn)

To any antisymmetric exchange matrix B one can associate a (coefficient-free) cluster algebra $A(B)$. We conjecture that $A(B)$ has a generic basis, and we prove this conjecture for a large class of cluster algebras arising naturally in Lie Theory. In particular, we obtain generic bases for all acyclic cluster algebras. (This confirms a conjecture due to Dupont.) This is joint work with Christof Geiss and Bernard Leclerc.

Strange duality of weighted homogeneous polynomials

Atsushi Takahashi (Osaka University)

We consider a mirror symmetry between invertible weighted homogeneous polynomials in three variables. We define Dolgachev and Gabrielov numbers for them and show that we get a duality between these polynomials generalizing Arnold's strange duality between the 14 exceptional unimodal singularities. This is a joint work with Wolfgang Ebeling.

Moduli spaces of stable quotients and the wall-crossing phenomena

Yukinobu Toda (University of Tokyo)

The notion of stable quotients is introduced by Marian-Oprea-Pandharipande, to give a compactification of the moduli space of maps from Riemann surfaces to the Grassmannian, which is different from stable map compactification. In this talk I will give a generalized notion of stable quotients which depends on a certain stability parameter, and show that stable quotients and stable maps are related by wall-crossing phenomena.

Some quotient algebras of affine Hecke algebras

Nanhua Xi (Chinese Academy of Sciences)

An affine Hecke algebra has a big center. By modulo central characters one gets some finite dimensional quotient algebras. We will discuss a few particular quotient algebras.

Hall type algebras associated to triangulated categories

Fan Xu (Tsinghua University)

We give a new proof of the theorem of Peng and Xiao which provides a way of constructing Lie algebras from 2-periodic triangulated categories. The proof supplies a new multiplication structure for indecomposable objects in a 2-periodic triangulated category.

Quiver mutations and derived equivalences

Dong Yang (University of Bonn)

I will talk about quiver mutations and the induced derived equivalences of certain differential graded algebras introduced by Ginzburg. I will present Keller's proof, which relate these equivalences to classical tilting theory via Calabi–Yau completions.

Minimal presentations for the fundamental groups of the complement of hyperplane arrangements

Masahiko Yoshinaga (Kyoto University)

The complement of the union of affine hyperplanes has special homotopy types, so called minimal CW complex. We shall discuss presentations of the fundamental groups arising from minimal CW decompositions of such spaces.

Monomorphism categories, cotilting theory, and Gorenstein-projective modules

Pu Zhang (Shanghai Jiao Tong University)

I'll talk about a relation between monomorphism category and cotilting theory, with applications.

The monomorphism category $\mathcal{S}_n(\mathcal{X})$ is introduced, where $\mathcal{X}$ is a full subcategory of the finitely generated module category A-mod of Artin algebra A. By a work of C. M. Ringel and M. Schmidmeier, one knows that $\mathcal{S}_n(A)$ is a functorially finite subcategory in morphism category $\mathrm{Mor}_n(A)$. Thus, $\mathcal{S}_n(A)$ has Auslander-Reiten sequences.

The key result of this paper establishes a relation between monomorphism category and cotilting theory: given a cotilting A-module, there is a canonical construction of a cotilting $T_n(A)$-module $\mathbf{m}(T)$, such that $\mathcal{S}_n({}^{\perp}T) = {}^{\perp}\mathbf{m}(T)$, where $T_n(A)$ is the upper triangular matrix algebra of A, and ${}^{\perp}T$ is the left perpendicular category of T. A proof needs the six adjoint pairs between $\mathcal{A}$ and $\mathrm{Mor}_n(\mathcal{A})$, and the contravariantly finiteness of $\mathcal{S}_n(A)$ in $\mathrm{Mor}_n(\mathcal{A})$.

This reciprocity of the operators $\mathcal{S}_n$ and ${}^{\perp}$ has several applications. First, $\mathcal{S}_n(\mathcal{X})$ is a resolving contravariantly finite subcategory in $T_n(A)$-mod with $\widehat{\mathcal{S}_n(\mathcal{X})} = T_n(A)$-mod if and only if $\mathcal{X}$ is a resolving contravariantly finite subcategory in A-mod with $\widehat{\mathcal{X}} = A$-mod.

As other applications, taking $T = A$ for Gorenstein algebra A, the category $T_n(A)$-$\mathcal{G}proj$ of Gorenstein-projective $T_n(A)$-modules can be explicitly determined as $\mathcal{S}_n(^\perp A)$. By D. Happel's triangle-equivalence $D^b(A)/K^b(\mathcal{P}(\mathcal{A})) \cong \underline{^\perp\mathcal{A}}$ for Gorenstein algebra A, one has the description $D^b(T_n(A))/K^b(\mathcal{P}(T_n(A))) = \underline{\mathcal{S}_n(^\perp A)}$ of the singularity category of $T_n(A)$. Also, self-injective algebras A can be characterized by the property $T_n(A)$-$\mathcal{G}proj = \mathcal{S}_n(A)$.

Inspired by M. Auslander's idea, by using the description $\mathcal{S}_n(A) = {}^\perp\mathbf{m}(D(A_A))$, it is proved that $\mathcal{S}_n(A)$ is of finite representation type if and only if there is a bi-generator M of $\mathcal{S}_n(A)$ such that $\operatorname{gl.dim}\operatorname{End}_A(M) \le 2$. As a consequence, for a self-injective algebra A, $T_n(A)$ is of finite Cohen-Macaulay type if and only if there is a $T_n(A)$-generator M which is Gorenstein-projective, such that

$$\operatorname{gl.dim}\operatorname{End}_{T_n(A)}(M) \le 2.$$